THE REFLECTION OF SV-WAVES IN A POROELASTIC HALF-SPACE SATURATED WITH VISCIOUS FLUID

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ABSTRACT
In this paper, the effect of finite skeleton permeability of soils on the surface strains, rocking strains, and energy partitions during the reflection of SV-waves in a poroelastic half-space saturated with viscous fluid is presented. The motion of the medium of the half-space is described using Biot’s theory of wave propagation in fluid-saturated porous media. Numerical results presented in this paper show that the skeleton permeability of soils has a negligible effect on the rocking strains with respect to vertical displacement. The effect of permeability on the surface strains and rocking strains with respect to horizontal displacement becomes noticeable as the value of porosity increases and the value of Poisson’s ratio decreases. In general, the skeleton permeability of soils decreases the energy carried by the slow P-wave and increases the energy carried by the SV-wave.

1. INTRODUCTION
In this paper, the seepage force due to soil permeability and fluid viscosity is included in the analysis to show its effect on the surface strains, rocking strains, and energy partitioning in a poroelastic half-space during the reflection of SV-waves. Many researchers have studied the reflection of plane waves and surface waves from the surface of a poroelastic half-space. Biot published a series of papers on the wave propagation of elastic waves in a poroelastic half-space saturated by a viscous, compressible fluid in the lower frequency range and the higher frequency range [1, and 2] and predicted two dilatational P-waves (fast and slow) and one rotational S-wave. Also, Biot [3, and 4] presented an extension of his theory of wave propagation in a poroelastic half-space saturated by a viscous, compressible fluid to anisotropic media and porous dissipative media. Also, Deresiewicz and colleagues published a series of papers on the effects of boundary conditions on wave propagation in a fluid-filled porous solid, such as: the reflection of plane waves in a fully saturated half-space with a non-dissipative fluid [5], Love waves in a porous layer saturated with a viscous fluid [6], the reflection of plane waves in a fully saturated half-space with a viscous fluid [7], surface waves in a poroelastic half-space saturated with a viscous fluid [8], and reflection of plane waves at an irregular boundary [9]. The well known results on the reflection of plane P-waves
and SV-waves from a fully saturated half-space with inviscid fluid are obtained by Lin et al [10, 11, and 12]. Y. S. Al Rjoub [13] has studied the reflection of P-waves in a poroelastic half-space saturated with viscous fluid. He found that the effect of permeability of soils is negligible in the case of soft soils, while its effect is noticeable in the case of stiff soils. Many researchers have studied the surface strains, rocking strains, and energy partitioning in elastic media [14, 15, 16, 17, 18, and 19].

This paper follow Biot’s theory of wave propagation in a poroelastic half-space saturated by viscous fluid to study the surface strains, rocking strains, and energy partitioning during the reflection of an incident SV-wave. The results obtained in this paper are compared with those obtained by Lin et al [10, 11, and 12] for a fully saturated half-space in which the effect of permeability of soils and viscosity of fluid is negligible.

2. THE WAVE EQUATIONS OF BIOT’S THEORY IN A POROELASTIC HALF-SPACE

Biot’s theory of wave propagation in a poroelastic half-space is used to describe the motion of the medium of the half-space, in which the displacement vector of the solid skeleton $\vec{u}$ and the displacement vector of the pore fluid $\vec{U}$ satisfy the following two coupled equations of motion (1,3):

$$\mu \nabla^2 \vec{u} + \text{grad}[\lambda \varepsilon + Q \varepsilon] + 2b \frac{\partial}{\partial t}(\rho_{11} \vec{u} + \rho_{12} \vec{U}) = \frac{\partial^2}{\partial t^2} \left( \rho_{11} \vec{u} + \rho_{12} \vec{U} \right) \frac{\partial}{\partial t} (\vec{u} - \vec{U})$$

$$\text{grad}[Qe + R \varepsilon] = \frac{\partial^2}{\partial t^2} \left( \rho_{11} \vec{u} + \rho_{12} \vec{U} \right) - b \frac{\partial}{\partial t} (\vec{u} - \vec{U})$$

where $\varepsilon = \text{div}(\vec{u}), \varepsilon = \text{div}(\vec{U})$; $(\lambda, \mu, Q, R)$ are the material constants of the mixture; $b$ is the dissipation coefficient, which depends on the skeleton permeability($\hat{k}$), the fluid viscosity($\hat{\mu}$), and the soil porosity($\hat{n}$) through $b = \hat{n}^2 \hat{k}$; and $(\rho_{11}, \rho_{12}, \rho_{22})$ are the dynamic mass coefficients.

The displacement vectors of the solid-skeleton $\vec{u}$ and those of the pore fluid $\vec{U}$ can be written as:

$$\vec{u} = \text{grad} \phi + \text{curl} \psi$$

$$\vec{U} = \text{grad} \Phi + \text{curl} \Psi$$

where $\phi$ and $\psi$ are the P-wave and SV-wave potentials of the solid motion, and $\Phi$ and $\Psi$ are the P-wave and SV-wave potentials of the fluid motion.

As a result of the Helmholtz decomposition of equation (2), a harmonic solution of the equations of motion is obtained, which consists of two P-waves (fast and slow) and one SV-wave. Then, the wave velocities $V_p$ and potentials $f_p$ of the two P-waves (fast and slow) and those of the SV-wave can be calculated as:

$$V_{p,j} = \sqrt{\frac{2A}{B \pm (B^2 - 4AC)^{1/2}}}, j = f, s$$

$$f_{p,j} = \frac{A/ V_{o,j}^2 - \rho_{11} R + \rho_{12} Q \pm \left( \frac{ib}{\omega} \right) (Q + R)}{\rho_{12} R - \rho_{22} Q \pm \left( \frac{ib}{\omega} \right) (Q + R)}, j = f, s$$

$$V_{sv} = \sqrt{\frac{2A}{C}}$$
\[ f_{sv} = -\left( \frac{\rho_{12} + i\hat{b}/\omega}{\rho_{22} - i\hat{b}/\omega} \right) \]  
(6)

where \( A = PR - Q^2 \)

\[ B = \rho_{12}R + \rho_{22}P - 2\rho_{12}Q - \frac{i\hat{b}}{\omega}(P + R + 2Q) \]

\[ C = \rho_{11}\rho_{22} - \rho_{12}^2 - \frac{i\hat{b}}{\omega}(\rho_{11} + \rho_{22} + 2\rho_{12}) \]

2.1 Material constants for porous media

Simplified formulas for the three dynamic mass coefficients \((\rho_{11}, \rho_{12}, \rho_{22})\) and the four elastic moduli \((\lambda, \mu, Q, R)\) are used, following [10]:

\[ \rho_{11} = (1 - \hat{n})\rho_g - \rho_{12} \]  
(7)

\[ \rho_{12} = \hat{n}(\tau_a - 1)\rho_f \]  
(8)

\[ \rho_{22} = \hat{n}\rho_f - \rho_{12} \]  
(9)

\[ \mu = \mu_s \]  
(10)

\[ \lambda = \lambda_s + Q^2 / R \]  
(11)

\[ Q = (1 - \hat{n})K_f \]  
(12)

\[ R = \hat{n}K_f \]  
(13)

where \( \lambda_s \) is the Lame constant for the solid-skeleton which depends on the shear modulus of the skeleton \((\mu_s)\) and Poisson's ratio of the skeleton \((\nu_s)\) through \( \lambda_s = \frac{2\nu_s\mu_s}{1 - 2\nu_s} \); \( K_f \) is the bulk modulus of the fluid; \( \rho_g \) is the density of the solid; \( \rho_f \) is the density of the fluid; and \( \tau_a \) is the dynamic tortuosity, taken as 0.5 in this paper, assuming pores are formed by spherical grains.

In this paper, water is assumed to be the pore fluid, which has a mass density of \( 10^3 \text{kg/m}^3 \), a bulk modulus of \( 2.2 \times 10^9 \text{Pa} \), and a viscosity of \( 10^{-3} \text{Pa.s} \). The mass density of the grains is \( 2.7 \times 10^3 \text{kg/m}^3 \). The ratio \( K_f / \mu \) (1.0 and 100) is used in this paper, which represents the fluid bulk modulus with respect to the stiffness of the solid material. Two different values of Poisson’s ratio are used, 0.1 and 0.4. Also, two different values of porosity are used, 0.3 and 0.1. Two different values of intrinsic permeability of the skeleton, in units of \( \text{m}^2 \) are used, \( k=10^{-6} \) and \( k=10^{-10} \). A frequency not larger than 100 Hz is considered in this paper to ensure that laminar flow through the pores occurs. It should be noted that the physical properties of the materials used in this study fall in the range where Biot’s theory of wave propagation does apply (grain size, ASTM soil classification, effective pore size, permeability, frequency range, etc.).

3. INCIDENT SV-WAVE

An incident SV-wave with incidence angle \( \theta_{sv} \) interacting with a fluid-saturated half-space, as shown in Figure 1, can be represented by its potential as:

\[ \psi' = K_0 \exp[i k_{sv}(x \sin \theta_{sv} - y \cos \theta_{sv}) - i \omega t] \]  
(14)
Two dilatational P-waves (fast and slow) and one rotational SV-wave will be generated as a result of the interaction between the incident SV-wave and the free-surface of the half-space and can be represented by their potentials as:

\[ \phi_f' = K_{pf} \exp[i k_f (x \sin \theta_{pf} + y \cos \theta_{pf}) - i \omega t] \]  

\[ \phi_s' = K_{ps} \exp[i k_s (x \sin \theta_{ps} + y \cos \theta_{ps}) - i \omega t] \]  

\[ \psi_r' = K_{sv} \exp[i k_{sv} (x \sin \theta_{sv} + y \cos \theta_{sv}) - i \omega t] \]

where \( K_{0}, K_{pf}, K_{ps}, K_{sv} \) are the amplitude coefficients of the incident SV-wave, reflected fast P-wave, reflected slow P-wave, and reflected SV-wave, respectively; \( k_f, k_s, k_{sv} \) are the complex wave numbers of the fast P-wave, slow P-wave, and SV-wave, respectively.

Two critical angles are formed since two P-waves co-exist in a poroelastic half-space. The first critical angle is for the fast P-wave, \( \theta_{cr1} = \sin^{-1}\left(\frac{V_{sv}}{V_{pf}}\right) \), and the other critical angle for the slow P-wave, \( \theta_{cr2} = \sin^{-1}\left(\frac{V_{sv}}{V_{ps}}\right) \). The reflected P-waves become surface waves once the incident angle reaches the critical angle.

The four amplitude coefficients are taken from Ref. [10] for the case where permeability of soils is neglected and from Ref. [20] for the case where permeability of soils is included in the analysis.

The surface strains can be written as:

\[ \begin{align*}
\gamma_x' &= \frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial x \partial y'} \\
\gamma_y' &= \frac{\partial^2 \phi'}{\partial y^2} + \frac{\partial^2 \psi'}{\partial x \partial y'}
\end{align*} \]

The surface rocking can be determined following [21, 14]:

\[ \psi_{xy} = -\frac{1}{2} \left( \frac{\partial^2 \psi'}{\partial x^2} - \frac{\partial^2 \psi'}{\partial y^2} \right)_{y=0} = \frac{1}{2} \left( K_0 + K_{sv} \right) k_{sv} \exp(i k_{sv} \mu) \psi_{xy} \]

The surface rocking is normalized by the SV-wave number, which can be expressed following [14] as,

\[ \xi_{xy} = -(K_0 + K_{sv}) \exp(i \pi/2) \]

The energy partitions for each wave potentials can be calculated following [22] as,

\[ E = \left| \tilde{T} \tilde{u} \right| \]

4. EFFECT OF PERMEABILITY ON THE SURFACE STRAINS, ROCKING, AND ENERGY PARTITIONS

4.1 Effect of permeability on the surface strains

Figure 2 and Figure 3 show the horizontal and vertical surface strain amplitudes for a poroelastic half-space saturated with viscous fluid. In these figures, the left-hand side shows the surface strain amplitudes with \( \frac{K_f}{\mu} = 1 \), while the right-hand side shows the surface strain.
amplitudes with \( \frac{K_f}{\mu} = 100 \). Two values of permeability are used in each figure, k=10^{-6} m^2 and k=10^{-10} m^2. The results demonstrate that: as the value of \( \frac{K_f}{\mu} \) decreases, all cases with seepage force match those with no seepage force, especially as the value of Poisson’s ratio increases; as the porosity increases, both vertical and horizontal strain amplitudes decrease; as the value of \( \frac{K_f}{\mu} \) increases, the effect of permeability becomes noticeable, especially at low values of Poisson’s ratio. As the value of permeability increases, say k=10^{-6} m^2, it matches the results for the case of no seepage force. The peak surface strains in the x- and y-directions, which occurs at the first critical angle, increase as the value of permeability decreases.

4.2  Effect of permeability on rocking

Figure 4(A, B) and Figure 5 show the ratio of rocking angles to both horizontal and vertical displacement for a poroelastic half-space saturated with viscous fluid, respectively, with different values of \( \frac{K_f}{\mu} \), different porosities, and different values of Poisson’s ratio. In these figures, the left-hand side shows rocking with respect to horizontal and vertical displacement with \( \frac{K_f}{\mu} = 1 \), while the right-hand displays results for \( \frac{K_f}{\mu} = 100 \). Also, two values of permeability are used, k=10^{-6} m^2 and k=10^{-10} m^2.

It can be seen from the results that the ratio of rocking angle with respect to horizontal displacement increases as the value of the permeability decreases. The ratio of rocking angles with respect to horizontal displacement goes to infinity at 45° because the horizontal displacement is equal to zero at that angle for all cases. The ratio of rocking angle with respect to vertical displacement is equal to 2\sin\theta for all cases, see Figure 5. The ratios of rocking angles with respect to vertical displacement for porosity equal to 0.1 are not shown for this reason. Figure 6(A, B) shows the normalized rotation. The left-hand side of the figure shows the normalized rotation with \( \frac{K_f}{\mu} = 1 \), while the right-hand side shows the rotations with \( \frac{K_f}{\mu} = 100 \). Again, two values of permeability are used, k=10^{-6} m^2 and k=10^{-10} m^2. The effect of permeability becomes significant for the case of soft soil with low values of Poisson’s ratio and high porosity (compare upper two figures in 6A with upper two figures in 6B). The peak value of the normalized surface rocking is equal to 2 for all cases with an incident angle of 45°.

4.3  Effect of permeability on energy partitions

Figures 7, 8, and 9 show the energy rate for the reflected fast P-wave, reflected slow P-wave, and the reflected SV-wave for a poroelastic half-space saturated with viscous fluid with two different values of \( \frac{K_f}{\mu} \) (\( \frac{K_f}{\mu} = 1 \) in the left sides of the figures, \( \frac{K_f}{\mu} = 100 \) in the right-hand sides of the figures), different porosities, and different values of Poisson’s ratio. Again, two values of permeability are used, 10^{-6} m^2 and 10^{-10} m^2. The energy carried by the reflected fast P-waves is almost zero for the two cases with and without seepage force, especially for soft soils. The energy carried by the slow P-waves decreases as the value of Poisson’s ratio
increases (compare upper and lower figures of 8). The results show that, as the value of permeability decreases, the energy carried by the reflected slow P-waves decreases and that of the reflected SV-wave increases, regardless of the value of porosity (compare figures 8 and 9). The energy carried by the reflected SV-wave increases as the value of Poisson’s ratio increases for all cases (compare upper and lower figures of 9).

5. CONCLUSIONS

The effect of the skeleton permeability of soils on the surface strains, rocking strains, energy partitions during the reflection of SV-waves on a poroelastic half-space saturated with viscous fluid is carried out in this paper. Biot’s theory of wave propagation is used to describe the motion of the medium of the half-space. It is concluded that decreasing permeability affects the surface strains in both directions, especially at lower values of Poisson’s ratio, causing the peak strain in both directions to increase. The permeability has a small effect on the ratio of rocking angles with respect to horizontal displacement, while it has no effect on the ratio of rocking angles with respect to vertical displacement. Last, it can be concluded from this study that the permeability affects the energy carried by the reflected slow P-waves and SV-waves, i.e. as the permeability values decrease, the energy carried by reflected slow P-waves decreases while that of reflected SV-wave increases.

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REFERENCES


Figure 1 Poroelastic half-space saturated with viscous fluid subjected to an incident SV-wave.

Figure 2 Horizontal surface strain amplitudes for porosity = 0.3, a) $\frac{K_f}{\mu} = 1$ and b) $\frac{K_f}{\mu} = 100$. 
Figure 3 Vertical surface strain amplitudes for porosity = 0.3, a) $\frac{K_f}{\mu} = 1$ and b) $\frac{K_f}{\mu} = 100$.

Figure 4A) Ratio of rocking angles with respect to horizontal displacement for porosity = 0.3, a) $\frac{K_f}{\mu} = 1$ and b) $\frac{K_f}{\mu} = 100$. 
Figure 4B) Same as Figure 4A) but with porosity = 0.1.

Figure 5) Ratio of rocking angles with respect to vertical displacement for porosity = 0.3, a) $\frac{K_f}{\mu} = 1$ and b) $\frac{K_f}{\mu} = 100$. 
Figure 6A) Surface rocking for porosity = 0.3, a) $\frac{K_f}{\mu} = 1$ and b) $\frac{K_f}{\mu} = 100$.

Figure 6B) Same as Figure 6A) but with porosity = 0.1.
Figure 7 Energy partition of the reflected fast P-wave for porosity = 0.3, a) \( \frac{K_f}{\mu} = 1 \) and b) \( \frac{K_f}{\mu} = 100 \).

Figure 8 Energy partition of the reflected slow P-wave for porosity = 0.3, a) \( \frac{K_f}{\mu} = 1 \) and b) \( \frac{K_f}{\mu} = 100 \).
Figure 9 Energy partition of the reflected SV-wave for porosity = 0.3, a) $\frac{K_f}{\mu} = 1$ and b) $\frac{K_f}{\mu} = 100$. 