REDUCED ORDER MODELING FOR THE STATIC AND DYNAMIC GEOMETRIC NONLINEAR RESPONSES OF A COMPLEX MULTI-BAY STRUCTURE

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Keywords: reduced order modeling, nonlinear geometric response, finite elements.

ABSTRACT

This paper focuses on the continued development and deepened validation of nonlinear reduced order models of structures experiencing large deformations. Of particular interest here is a complex structure with rich dynamics: a previously introduced 9-bay panel with stiffeners and longerons modeled by finite elements using approximately 96,000 degrees-of-freedom. Building on a general methodology successfully validated in recent years on simpler beam and plate structures, a reduced order model of the panel motions is developed step-by-step. This 85-mode model is shown by comparison with full finite element (Nastran) results to lead to accurate predictions of both static and dynamic responses of the panel. Coupling of this reduced order model with piston-theory aerodynamics is also achieved to demonstrate the capability of these reduced order models to support multidisciplinary analyses.

1. INTRODUCTION

Significant efforts have centered in the last decade or so on the construction of reduced order models (ROMs) of structures undergoing “large” deformations, i.e. exhibiting geometric nonlinearity, from finite element models generated using commercial codes (e.g. Nastran, Abaqus, ANSYS), see [1]. These efforts aim at combining the strengths of commercial finite elements codes, i.e. vast library of elements, boundary conditions, and material definitions, with those of reduced order models, i.e. computational efficiency and ease of coupling with other discipline codes. The combination of these advantages is necessary in particular to address the full scale design of reusable hypersonic vehicles which will be subjected to strong aerodynamic, thermal, and acoustic excitations and thus will undergo large dynamic structural displacements inducing fatigue. In such problems, the structural deformations will feedback to the aerodynamics which will in turn affect the heat transfer to the panel implying the need to solve the structural-aerodynamic-thermal problems in a coupled multidisciplinary framework. Validation efforts of these reduced order models (see [1] are references therein) have ranged from applications to flat structures, to moderately large motions of curved structures. Further, the coupling of these nonlinear structural reduced order models with aerodynamics, either full or reduced order model has also been successfully demonstrated. A similar coupling but of the structural dynamics and thermal aspects, the two in reduced order model format, has also been
proposed and validated. Finally, a first assessment of the predictive capabilities of reduced order models for cracked panels was performed in [2]. However, a re-examination of these key aspects is needed for more complex structural models, i.e. which exhibit more complex physical behavior and thus necessitate a larger number of modes for an accurate representation of their response, e.g. the 9-bay panel of [3]. This re-examination was the focus of the work presented in [4], where a new identification approach and a first reduced order model of the panel developed and validated successfully on static responses. The present effort extends the work of [4] in regards to basis functions leading to a reduced order model predicting accurately both static and dynamic responses of the 9-bay panel.

2. REDUCED ORDER MODEL GOVERNING EQUATIONS

The reduced order models (ROM) considered here are based on a representation of the nonlinear geometric response of the structure in the form

\[ \mathbf{u}(t) = \sum_{n=1}^{M} q_n(t) \psi_n^{(n)} \]  

where, \( \mathbf{u}(t) \) denotes the vector of displacements of the finite element degrees of freedom and \( \psi_n^{(n)} \) are specified, constant basis functions. As shown in [1], the governing equations for the time dependent generalized coordinates \( q_n(t) \)

\[ M_{ij} \ddot{q}_j + D_{ij} \dot{q}_j + K_{ij}^{(1)} q_j + K_{ij}^{(2)} q_i, q_j + K_{ij}^{(3)} q_i, q_j, q_p = F_i \].

Note in Eq. (2) that a linear damping term \( D_{ij} \dot{q}_j \) has been added to collectively represent various dissipation mechanisms. Further, \( M_{ij} \) denotes the elements of the mass matrix, \( K_{ij}^{(1)}, K_{ij}^{(2)}, K_{ij}^{(3)} \) are linear, quadratic, and cubic stiffness coefficients and \( F_i \) are the modal forces.

3. IDENTIFICATION OF ROM PARAMETERS

The identification of the ROM parameters is the process by which the stiffness coefficients of Eq. (2) are determined using the commercial (Nastran here) finite element model of the structure and the basis functions \( \psi_n^{(n)} \). Several strategies are possible to accomplish this task (see [1] for review) but the procedure recently derived in [4] will be used instead because of its much reduced computational effort, especially when the number of basis functions is large. This approach relies on the availability of the final tangent stiffness matrix of the structure under an imposed displacement and is briefly reviewed below.

The \( iu \) component of the reduced order tangent stiffness matrix can be derived from the cubic stiffness operator of Eq. (2) as

\[ K_{iu}^{(T)} = \left[ \frac{\partial}{\partial q_u} \left[ K_{iu}^{(1)} q_j + K_{iu}^{(2)} q_i, q_j + K_{iu}^{(3)} q_i, q_j, q_p \right] \right] = K_{iu}^{(1)} + \left[ K_{iju}^{(2)} + K_{iju}^{(2)} \right] q_j + \left[ K_{iju}^{(3)} + K_{iju}^{(3)} + K_{iju}^{(3)} \right] q_i, q_j, q_p \]
The stiffness coefficients $K^{(1)}_{ij}$, $K^{(2)}_{ij}$, and $K^{(3)}_{ijlp}$ can then be determined by imposing the matching, for a series of deformed configurations, of the reduced order tangent stiffness matrix with the projection on the basis of its finite element counterpart $\hat{\mathbf{K}}^{(T)}$. That is,

$$K^{(T)}(\mathbf{q}^{(p)}) = \Psi^T \hat{\mathbf{K}}^{(T)}(\mathbf{u}^{(p)}) \Psi$$

where $\mathbf{u}^{(p)} = \Psi \mathbf{q}^{(p)}$ for a series of $p = 1, \ldots, P$ deformed configurations. In the above equations, the subscript $^T$ denotes the operation of matrix transposition and $\Psi$ is the modal matrix

$$\Psi = \begin{bmatrix} \psi^{(1)} & \psi^{(2)} & \ldots & \psi^{(M)} \end{bmatrix}.$$  

The deformed configurations $\mathbf{u}^{(p)} = \Psi \mathbf{q}^{(p)}$ selected here are those of the imposed displacement scheme (see [1] and [5]). Consider first the situation in which the imposed displacement is along a single basis function, i.e. $\mathbf{u} = q_j \psi^{(j)}$. The corresponding ROM tangent stiffness matrix can then be written as (no sum on $j$)

$$K^{(T)}_{iu} = K^{(1)}_{iu} + \left[K^{(2)}_{iju} + K^{(2)}_{iuj}\right]q_j + \left[K^{(3)}_{iju} + K^{(3)}_{iju} + K^{(3)}_{iul}\right]q_j^2$$

Since the elements $K^{(2)}_{ijl}$ and $K^{(3)}_{ijlp}$ may be selected as zero unless $p \geq l \geq j$, the above equation is equivalent to

$$K^{(T)}_{iu} = K^{(1)}_{iu} + \left[K^{(2)}_{iju} + K^{(2)}_{jui}\right]q_j + K^{(3)}_{iju}q_j^2 \quad j < u$$

$$K^{(T)}_{iu} = K^{(1)}_{iu} + 2K^{(2)}_{iuu}q_u + 3K^{(3)}_{iuu}q_u^2 \quad j = u$$

$$K^{(T)}_{iu} = K^{(1)}_{iu} + K^{(2)}_{ijl}q_j + K^{(3)}_{ijl}q_j^2 \quad j > u$$

from which the coefficients $K^{(2)}_{ijl}$, $K^{(3)}_{ijl}$, and $K^{(3)}_{ijl}$ can be estimated if it is assumed that the linear stiffness coefficients are obtained as in linear modal analyses.

The final step in the identification of the reduced order model is to evaluate the coefficients $K^{(3)}_{iju}$ for $j \neq l$, $j \neq u$, and $u \neq l$. They can be evaluated from the knowledge of $K^{(T)}_{iu}$ corresponding to a displacement field which involves both basis functions $j$ and $l$, i.e. of the form of $\mathbf{u} = q_j \psi^{(n)} + q_l \psi^{(m)}$. Then, $K^{(T)}_{iu}$ is given by Eq. (3) in which no summation on $j$ and $l$ applies. Specifically, for $u > l > j$, one has

$$K^{(T)}_{iu} = K^{(1)}_{iu} + \left[K^{(2)}_{iju} q_j + K^{(2)}_{iju} q_j\right] + \left[K^{(3)}_{iju} q_j q_l + K^{(3)}_{iju} q_j^2 + K^{(3)}_{iju} q_l^2\right]$$

in which all terms are known except $K^{(3)}_{iju}$.
Note in the above procedure that only combinations of two modes are used and thus the number of deformed configurations to consider is only of order $O(M^2)$, it is indeed $2M + M(M - 1)/2$.

4. ROM BASIS SELECTION

One of the key challenges in the construction of a good reduced order model is in the selection of the basis functions $\psi^{(m)}$; if the structural response is not well represented within this basis, the corresponding prediction of the reduced order model will in general be poor. The modes/basis functions needed for a nonlinear problem are certainly expected to include those used for the corresponding linear problem, but others are also anticipated to model the difference in physical behavior induced by the nonlinearity not directly by the loading.

This issue was addressed in [6] through the inclusion in the basis of an additional set of basis functions referred to as dual modes aimed at these nonlinear effects. The key idea in this approach is to first subject the structure to a series of “representative” static loadings, and determine the corresponding nonlinear displacement fields. Then, extract from them additional basis functions, the “dual modes”, to append to the linear basis, i.e. the modes that would be used in the linear case. It was argued in [6] that the representative static loadings should be selected to excite primarily the linear basis modes, and, in fact, in the absence of geometric nonlinearity (i.e. for a linear analysis) should only excite these modes, i.e. the applied load vectors $F_{FE}^{(m)}$ on the structural finite element model should be such that the corresponding linear static responses are of the form

$$\sum_{i} \alpha_{i}^{(m)} \psi_{(i)} = \sum_{i} \alpha_{i}^{(m)} K_{FE}^{(i)} \psi_{(i)}^{(i)}$$

where $\alpha_{i}^{(m)}$ are coefficients to be chosen with $m$ denoting the load case number. A detailed discussion of the linear combinations to be used is presented in [9] but, in all validations carried out, it has been sufficient to consider the cases

$$F_{FE}^{(m)} = \alpha_{i}^{(m)} K_{FE}^{(i)} \psi_{(i)}^{(i)} \quad i = \text{dominant mode}$$

and

$$F_{FE}^{(m)} = \frac{\alpha_{i}^{(m)}}{2} K_{FE}^{(i)} \left[ \psi_{(i)}^{(i)} + \psi_{(j)}^{(j)} \right] \quad i = \text{dominant mode, } j \neq i$$

where a “dominant” mode is loosely defined as one expected to provide a large component of the panel response to the physical loading. The ensemble of loading cases considered is formed by selecting several values of $\alpha_{i}^{(m)}$ for each dominant mode in Eq. (13) and also for each mode $j \neq i$ in Eq. (14). Note further that both positive and negative values of $\alpha_{i}^{(m)}$ are suggested and that their magnitudes should be such that the corresponding displacement fields $\psi^{(m)}$ range from near linear cases to some exhibiting a strong nonlinearity.

The next step of the basis construction is the extraction of the nonlinear effects in the obtained displacement fields, which is achieved by removing from the displacements fields their projections on the linear basis. Finally, a proper orthogonal decomposition (POD) analysis of each set of “nonlinear responses” is then sequentially carried out to extract the dominant features of these responses which are then selected as dual modes, see [6] for full details.
5. DESCRIPTION OF VALIDATION MODEL

The 9-bay fuselage sidewall panel of [3] modeled within Nastran was considered for the validation of the novel (i) basis selection strategy and (ii) stiffness coefficients identification. The 9-bay panel is a section of the sidewall fuselage panel studied in [7], see Fig. 1 for a picture of the actual hardware taken from [7]. The finite element model of the 9-bay panel, shown in Fig. 2, has 96,156 degrees of freedom. The dimensions of the skin panel are 58.11in by 25.06in, and it is subdivided into nine bays by a riveted frame and longeron substructure. Each bay measures 18.75in by 7.5in between rivet lines. The thickness of the skin panel and frame substructure is of 0.05in and 0.04in for the longeron substructure. The finite element model consists of 4-node plate elements. Further, beam elements were used to model the rivets that join the skin panel to the frame and longeron substructures. The material properties are shown in Table 2. The edges of the skin panel are simply supported.

Figure 1. Sidewall fuselage panel taken from [7]. 9-bay panel is a section of this structure.

Figure 2. Finite element model of the 9-bay fuselage sidewall panel, (a) isometric view, (b) top view.
6. REDUCED ORDER MODEL OF THE 9-BAY PANEL

One of the complexities of the 9-bay panel considered here is in its high modal density, the structure has 89 linear modes in the [0,500]Hz frequency band. This presents a challenge in the construction of a compact basis; therefore, it becomes very important to identify the most important modes that are needed to represent the response of the structure. To this end, an acoustic excitation, consisting of an overall sound pressure level (OASPL) of 144dB, was applied to the skin panel and a series of 200 “snapshots” were obtained from the stationary part of the MSC/Nastran SOL 400 nonlinear dynamic response. The appropriateness of a basis to model the response can be assessed by the representation error \( \varepsilon_{rep} = \left\| \mathbf{u} - \mathbf{u}_{proj} \right\|/\|\mathbf{u}\| \), where \( \mathbf{u} \) is the static displacement field computed by the finite element code and \( \mathbf{u}_{proj} \) is its projection on the selected basis. The mean of the representation error \( \left\langle \varepsilon_{rep} \right\rangle \) was plotted as a function of the number of retained linear modes and the modes at which noticeable drops in this error occurred were recorded. This process led to the identification of a set of 49 linear modes, with natural frequencies ranging from 68Hz to 541Hz. The mean of the representation error of the dominant transverse displacement (T3 or \( z \) component) was equal to 0.33% for the skin panel and 0.5% for the frame-longeron substructure. Further, the mean representation errors for the in-plane component along the length of the skin panel (T1 or x component) were equal to 43% for the skin panel and 2.7% for the frame-longeron substructure. The mean representation errors for the component along the width (T2 or y component), which is the dominant “in-plane” component for the skin panel, were equal to 94% for the skin panel and 4.7% for the frame-longeron substructure. The large errors of the in-plane components of the skin panel are fully expected and result from the membrane stretching that occurs when the behavior of the panel is in the nonlinear regime, and which the linear basis cannot capture.

The 14 linear modes with the largest modal components, i.e. modes 1, 5, 6, 7, 9, 10, 13, 15, 16, 25, 26, 28, 46 and 50, were used to construct the dual modes. Since the modal component of mode 1 is much larger than the other ones, it was considered as the only dominant mode in Eqs. (13)-(14). The POD-based dual mode construction procedure highlighted above (see [9] for full details) was performed for the data obtained for mode 1 alone and each of the 13 combinations of mode 1 and another of the 13 largest responding modes. In each of these 14 situations, 10 different loading factors \( \alpha_i^{(m)} \) were used, half positive and half negative, and leading to peak deflections ranging from 1 to approximately 2.4 skin panel thicknesses. The remainders of these 140 deflections, after projection on the 49 linear modes identified above, were analyzed by POD. A key aspect of this approach is to select the POD modes that contain new information originating from the geometric nonlinearity. In this light, there are two main substructures of interest, the skin panel and the frames. For the skin panel, it is of interest to select the POD eigenvectors with largest eigenvalues and having a dominant in-plane component (T1 and T2). On the other hand, it can be seen from Fig. 2(a) that the frames are analogous to a cantilevered structure. Therefore, it is of interest to select the POD eigenvectors with largest eigenvalues and having dominant T3 components, which is in the tangential or axial direction of the frames. Thirty-six duals modes were identified this way.

6.1. Static Validation on the 9-Bay Panel

Having completed the reduced order model construction, it was desired to assess its predictive
capability in comparison with NX/Nastran. To this end, a loading of 0.6 psi, leading to a 2.5 thicknesses maximum skin panel deflection, was considered and shown in Figs. 3-10 are contour plots of the different displacement components. Note the excellent matching, both qualitatively and quantitatively, between reduced order model and NX/Nastran results. The norm errors of the former in comparison to the latter, for the skin panel degrees-of-freedom, were 1.2% for the transverse (T3) component, 3.5% for the in-plane T2 component, 45% for the other, smaller, in-plane component T1, and 5% for the in-plane magnitude (see also Table 2). Clearly, the matching of the dominant components, T3 and T2, is very good. On the other hand, the relative error of the T1 component is still rather large but it is clear from Figs. 5-10 that this component is much smaller than its T2 counterpart (as stated above).

The prediction errors for the frame substructure were equal to 1.3% for T3, 13% for T1, 8% for T2, and 12% for the magnitude of the in-plane displacements.

<table>
<thead>
<tr>
<th>Skin Panel</th>
<th>Frame-Longeron Substructure</th>
</tr>
</thead>
<tbody>
<tr>
<td>T3</td>
<td>0.33%</td>
</tr>
<tr>
<td>In-Plane Mag</td>
<td>54%</td>
</tr>
<tr>
<td>T2</td>
<td>94%</td>
</tr>
<tr>
<td>T1</td>
<td>43%</td>
</tr>
</tbody>
</table>

Table 3: Summary of Representation and Prediction Errors

Figure 3. Translation magnitude induced by a uniform pressure of 0.6 psi, NX/Nastran.

Figure 4. Translation magnitude induced by a uniform pressure of 0.6 psi, 85-mode ROM.
Figure 5. Magnitude of the in-plane displacement induced by a uniform pressure of 0.6 psi, skin panel only, NX/Nastran.

Figure 6. Magnitude of the in-plane displacement induced by a uniform pressure of 0.6 psi, skin panel only, 85-mode ROM.

Figure 7. In-plane displacement along T2 induced by a uniform pressure of 0.6 psi, skin panel only, NX/Nastran.
Figure 8. In-plane displacement along T2 induced by a uniform pressure of 0.6 psi, skin panel only, 85-mode ROM.

Figure 9. In-plane displacement along T1 induced by a uniform pressure of 0.6 psi, skin panel only, NX/Nastran.

Figure 10. In-plane displacement along T1 induced by a uniform pressure of 0.6 psi, skin panel only, 85-mode ROM.
6.2. Nonlinear Dynamic Validation on the 9-Bay Panel

The 9-bay panel was subjected to a uniform pressure on its top surface varying randomly in time as a white noise band-limited process in the frequency range [0, 500]Hz to simulate an acoustic loading. The acoustic excitation consisted of an overall sound pressure level (OASPL) of 144dB. Further, to permit a close comparison between the full finite element and ROM results, a simple Rayleigh damping model was adopted, i.e. for which the damping matrix is $D = \alpha M + \beta K$ with $\alpha = 7.55/s$ and $\beta = 5.6E-6s$. This selection led to damping ratios between 0.65% and 1% for all transverse modes in the excitation band. The excitation level considered led to a peak transverse displacement of approximately 2.5 skin panel thicknesses, clearly within the geometric nonlinear regime.

The computational effort required to obtain a long time history of the nonlinear dynamic response of the 96,156 degree-of-freedom 9-bay panel was found to be very large. For instance, the analysis of 100,000 time steps required approximately 700GB of scratch space and the analysis lasted 6 days using 4 cores. Therefore, four time histories of 100,000 time steps were analyzed in MSC/Nastran SOL 400, the power spectral densities computed, and their mean used for validation of the 85-mode ROM. Results were obtained at the middle point of bays 1, 2, and 5 and at point A (see Fig. 11) on one of the frames.

Shown in Figs. 12 and 13 are plots of the power spectral density of the transverse (T3) and in-plane (T1 and T2) responses of the middle point of bays 1 and 2 (see Fig. 2(b) for panel numbering). The T2 component is very small at the middle point of bay 5; therefore, only the power spectral density of the T3 and T1 components are shown in Fig. 14. Finally, the power spectral density of the transverse and in-plane components, at point A of the frame, is shown in Fig. 15. Clearly, the matching of the T2 and T3 components of the Nastran response is very good at all points. Further, the matching of the dominant T1 component at point A is excellent as well. In addition, the matching of the dominant peaks of the T1 component of the skin panel is very good.

While it might be appealing to focus solely on the skin panel for the construction of the duals, a careful observation of the response of the T1 and T3 components at point A discourages the pursuit of this option. Not only is the energy of both components large, but as seen in Fig. 15(b), there are two dominant modes in the response of the T1 component. These modes are 1 and 7; mode 1 is a global mode, however, the response of mode 7 is mostly localized to the frame where point A is contained. The large T1 response of the frames is expected to lead to a tangential displacement (T3 direction), which can be modeled with the appropriate dual.

![Figure 11. Location of selected frame node for output of results.](image-url)
Figure 12. Power spectral density of the transverse (T3) and in-plane (T1 and T2) deflections at the middle point of bay 1. Reduced order model and finite element (“SOL 400”), SPL =144dB.
Figure 13. Power spectral density of the transverse (T3) and in-plane (T1 and T2) deflections at the middle point of bay 2. Reduced order model and finite element (“SOL 400”), $SPL = 144\text{dB}$. 
Figure 14. Power spectral density of the transverse (T3) and in-plane (T1) deflections at the middle point of bay 5. Reduced order model and finite element (“SOL 400”), SPL = 144dB.

Besides computational efficiency, another strong advantage of reduced order models is their ease in coupling with other discipline codes, especially other reduced order models, at the contrary of full order models. In the present context, coupling with the aerodynamics is of primary importance because of the loading and potential heating it induces on the structure, with the potential of instability (e.g. flutter, buckling) they create. The consideration of heat convection to the structure requires the modeling of its temperature distribution and the associated coupling with the structural deformations. These latter efforts can also be achieved within a reduced order modeling framework as demonstrated for example in [8-12] but will not be addressed here further leaving the loading as the sole effect of aerodynamics on the structure. To demonstrate the coupling, the aerodynamic was represented through a third order piston theory with the flow assumed along the $x$ direction, see Fig. 2, on the top of the panel (i.e. the side opposite to the longerons). Further, the altitude and Mach number were assumed to be 40,000ft and 1.6 which is below flutter speed. The aerodynamic force was included in the time marching of Eq. (2) and shown in Figs 12-15 are the corresponding spectra of the displacements induced on the aeroelastic system by the acoustic excitation. Comparing the responses with and without aerodynamics, it is seen that the latter induces here
Figure 15. Power spectral density of the transverse (T3) and in-plane (T1) deflections at point A of the frame. Reduced order model and finite element ("SOL 400"), SPL =144dB.
a reduction of the response which is consistent with the flow being below flutter speed. The computations can similarly be carried out above flutter speed as well. In such cases, the structural basis must also include all of the linear modes required for an accurate prediction of the linear flutter speed and mode in addition to those dictated by the acoustic loading.

7. SUMMARY

The focus of this investigation was on the continued development and validation of nonlinear reduced order models for structures experiencing large deformations. The 9-bay panel considered here is by far the most complex structure ever considered in such validation efforts. The construction of the 85-mode basis to represent the structural response was performed first, using linear and dual modes. The identification of the coefficients of this model was accomplished next using a recently proposed algorithm particularly focused on such large models. Comparisons with full finite element results demonstrated that the resulting reduced order model provides very accurate predictions of both static and dynamic responses of the panel in significantly nonlinear situations, e.g. 2.5 thicknesses peak displacements. Finally, the coupling of the structural reduced order model with aerodynamics was demonstrated in a simple case in which the latter was modeled using third order piston theory.

ACKNOWLEDGMENTS

The financial support of this work by the grant FA9550-10-1-0080 from the Air Force Office of Scientific Research with Dr D. Stargel as grant monitor is gratefully acknowledged. In addition, the authors wish to express their appreciation to Drs. S.A. Rizzi and A. Przekop for the use of the 9-bay finite element model.

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