TRANSITION AMONG VEERING, CROSSING AND LOCK-IN THROUGH VARIATION OF THE SYSTEM PARAMETERS

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ABSTRACT

The phenomena of mode veering, crossing and lock-in are analyzed in this paper. Their occurrence is generally found, under different conditions, when there is a parameter varying in the system, which produces a change in its behavior. It often happens that, when the frequencies approach each other, they can cross, veer and eventually present a lock-in state. The problem is analytically investigated for a general weakly-coupled two-degrees of freedom systems and a simulation example of two coupled beams is presented. Moreover, numerical and experimental evidences of lock-in in brake squeal are recalled to show how in gyroscopic systems, when two coupled mechanical parts have the same eigenvalues (lock-in state) then the whole system may becomes unstable and at that frequency and noise is generated. How things change when the coupling is not weak is address, and is also discussed to highlight phenomena of particular interest.

1. INTRODUCTION

In real structures, uncertainties, irregularities or variable operative conditions may affect significantly the behavior of similar structures. This problem is widely considered in the literature either theoretically or experimentally. Here we examine under which conditions may particular types of coupling between the system degrees of freedom (elastic, inertial or gyroscopic) lead to eigenvalues interaction, producing often interesting and unexpected phenomena. We refer specifically to three main situations, i.e. veering, crossing and mode lock-in, that are primarily related to the modes of the system but become a singular signature of the whole system behavior.

Variations in the geometric and physical parameters (e.g. stiffness, mass, friction coefficients, etc.) due to design variations, ware-off or operational conditions change the system eigenvalues. Occasionally, a pair of eigenvalues can approach each other and, when this happens, small variations in those parameters can lead to dramatic changes in the whole system dynamics. For example, small variations in gyroscopic terms may induce the transition between mode veering and mode lock-in in a brake system, causing squeal noise.
Mode veering is a common phenomenon associated with the eigenvalue loci: due to the variation of one or more parameters, two modes may approach each other but, instead of crossing, they veer away and finally diverge. The outstanding observation is that, after veering, the two corresponding eigenvectors interchange, i.e. the direction of the eigenvalue locus of one mode corresponds to the direction of the other mode before veering.

Mode crossing can be considered a particular and quite unusual case of veering. In linear conservative and non gyroscopic systems, the weaker the elastic coupling between the degrees of freedom is, the smaller the range of parameters is where veering occurs. When the off-diagonal terms of the stiffness matrix are zero, it is possible that the two uncoupled system have the same frequency and, thus, crossing is observed, i.e. the natural frequencies of two modes exchange their position in frequency. In this case, at the crossing point, the two modes are not uniquely defined, and can be described as the resultant of the two independent eigenvectors approaching the crossing point. Quite often, for weakly coupled systems, veering is mistaken by the more intuitive concept of crossing.

More generally, crossing can be observed in non conservative systems. In fact, in presence of high damping, two modes can have the same natural frequency though maintaining their individual mode shapes.

Mode lock-in is a phenomenon mirror to veering, that can be observed when gyroscopic terms are present. Due to the variation of one or more parameters, the frequencies of two modes approach more and more but, instead of veering away, they are attracted to one another and coalesce, leading to an unstable behavior.

To illustrate these phenomena, in Figure 1 three cases are qualitatively depicted for two natural frequencies $f_n$ versus a parameter variation $p$ (e.g. coupling stiffness). Three similar systems, characterized by veering, crossing and lock-in, respectively, are considered. It is worthwhile to point out that, far from the critical zone, the three systems have similar dynamics, while, inside this zone, the behaviors, qualitatively represented in the figure, show quite large differences. It is important to stress here that the width of the critical zone depends on the coupling between the DOFs of the systems.

![Figure 1: Veering, crossing and mode lock-in](image)

Several works have shown the experimental evidence of these phenomena. Veering is certainly the one by far the most investigated. After some preliminary observations on curved plates and beams [1,2], Leissa [3] gave a detailed description of it and named it veering. In [4], Perkins and Mote formulated a first analytical criterion for veering, while Pierre [5] developed a perturbation technique to show that the occurrence of strong mode localization and eigenvalue loci veering are manifestations of the same phenomenon, often caused by small structural irregularities in systems with close eigenvalues. Other theoretical and numerical works on veering are due to Balmes [6], du Bois et al. [7], Bonisoli et al. [8]. Vidoli and Vestrioni [9] gave recently a geometrical formulation of veering, and some experimental evidence of it are presented in [9, 10]. In [9] the case of a plate embedded with a set of piezoelectric actuator is considered, while in [10] Lin and Parker discuss the onset of
veering in planetary gears. Very recently Mace made a clear picture of the veering phenomenon in weakly coupled systems [11].

Crossing has less evidence in the literature, although any theoretical work on veering discusses in principle the crossing phenomenon (e.g. [4 – 9]). However in the authors’ knowledge there are not specific descriptions of experimental crossing phenomena, excluding those that were appropriately designed to show this possibility (see e.g. [7]).

Mode lock-in (also called modal coupling instability or flutter instability) was particularly observed in flutter phenomena and several problems of aerodynamic instability, and, more recently, in brake squeal noise, in several types of problems of (especially dry) contact between solid bodies [12] and in gyroscopic systems with negatively definite stiffness matrix [13]. Considering specifically contact problems, mode lock-in was initially highlighted in the beam-on disk set-up, developed by Akay at al. to investigate friction at interfaces [14]. More recent studies on brake squeal noise with simplified lab set-ups have always shown that squeal noise is an instability condition, that is reached when two eigenfrequencies of the system, due to the asymmetry of the stiffness matrix caused by friction forces, coalesce and become unstable [15-18].

Goal of this paper is to show and discuss under which conditions veering, crossing and mode lock-in can be observed, and to show some numerical and experimental cases of these phenomena.

2. ANALYTICAL DEVELOPMENTS

2.1 Veering

Let us first consider the rather simple two degrees of freedom undamped system, depicted in Figure 2.

![Figure 2: two-DOF System model](image)

Without loss of generality, let us consider the case where \( m_1 = m_2 = 1 \) so that the equation of motion is:

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
k + \varepsilon & -\varepsilon \\
-\varepsilon & k_1 + \varepsilon
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

where the stiffness \( \varepsilon \) between the two masses governs the coupling between the degrees of freedoms \( x_1 \) and \( x_2 \).

The eigenvalues \( \lambda_1 \) and \( \lambda_2 \) of this system are:

\[
\lambda_{1,2} = \frac{k + k_1 + 2\varepsilon \pm \sqrt{\Delta}}{2}, \Delta = (k + k_1 + 2\varepsilon)^2 - 4(k + \varepsilon)(k_1 + \varepsilon - \varepsilon^2) = (k - k_1)^2 + 4\varepsilon^2
\]

It is always \( \Delta > 0 \) implying that the system has always two distinct eigenvalues, and the minimum distance between the eigenvalues is reached when \( k = k_1 \), and its value is \( 2\varepsilon \). In other words the coupling term governs the minimum distance between the eigenvalues of the system.

By running a parametric analysis, keeping \( k = 1 \) N/m and \( \varepsilon = 0.01 \) N/m constant, and varying \( k_1 \) in the range \( 0.9 \) N/m \(< k_1 < 1.1 \) N/m, one has the classical eigenvalue veering (Figure 3a).
By increasing the coupling to $\varepsilon = 0.1 \text{ N/m}$, one obtains the results presented in Figure 3b, note that the two plots in Figure 3 have the same qualitative trend, but the scales in the axis are different. In a two degrees of freedom system, where only two eigenvalues are interacting, the coupling does not really cause any qualitative changes. However, in more complex systems (more degrees of freedom) this is not always the case, because the larger is the coupling the more the eigenvalues interact with each other, leading to more complex dynamics.

![Figure 3: Veering in an undamped non gyroscopic two-DOF system: a) weakly-coupled system; b) strongly-coupled system.](image)

Note that, for the two-DOF system, the case of crossing occurs only when $\varepsilon = 0$, that is the trivial case of two distinct single degrees of freedom systems.

In mode veering, the mode shapes associated with the eigenvalues morph as the natural frequencies veer apart. In particular, before and after the veering, two recognizable mode shapes are present. However, the two shapes “exchange” their natural frequencies, i.e. the lower frequency mode before veering becomes the higher frequency mode after veering. During the veering, both modes present a more complex not recognisable mode shape.

This can be quantified using the Modal Assurance Criterion (MAC) between the initial mode shape and the mode shapes that appear during veering.

Let us define the function:

$$mac(k_i)_{ij} = MAC\{\Phi_i(k_{in}), \Phi_j(k_i)\}$$

(3)

where the MAC operator is defined as:

$$MAC(\Phi, \Psi) = \frac{|\Phi^\top \Psi^*|^2}{(\Phi^\top \Phi^*) (\Psi^\top \Psi^*)}$$

(4)

The first index $i$ is the order of the $i^{th}$ ($i = 1, 2$) mode $\Phi_i$ before veering computed for the initial value of $k_i = k_{in}$, while the $j^{th}$ index ($j = 1, 2$) refers to the $j^{th}$ mode $\Psi_j(k_i)$ of the system during the veering as $k_i$ increases. In Figure 4 the four mac functions are plotted for $k_i$ ranging between 0.85 N/m and 1.15N/m.

![Figure 4: Mac between modes across the veering.](image)
In this simple case, the two shapes switch position at \( k = k_{cr} = 1\text{N/m} \). In fact, above this value, \( \text{mac}_{11} < \text{mac}_{12} \), i.e. the MAC of mode 1 with itself is lower than the MAC between modes 1 and 2, and the same holds for mode 2, i.e. \( \text{mac}_{22} < \text{mac}_{21} \). During veering, for \( 0.97\text{N/m} < k < 1.03\text{N/m} \), the low level of the four mac functions detects the presence of mode shapes that are no longer similar to the original ones.

### 2.2. Lock-in

Let us now add a gyroscopic term to the two-DOF system, so that the system under consideration is governed by equation (5).

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
k + \varepsilon & -\varepsilon + \alpha \\
-\varepsilon + \alpha & k_1 + \varepsilon
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

(5)

This equation is often used in literature to describe, with a low-order model, the unstable dynamics of brake squeal noise [15], or the aerodynamic flutter instability.

In this case, the eigenvalues of the system may be complex; they are given by:

\[
\lambda_{1,2} = \frac{k + k_1 + 2\varepsilon \pm \sqrt{\Delta}}{2}, \Delta = (k - k_1)^2 + 4(\varepsilon^2 - \alpha^2)
\]

(6)

For a fixed value of \( \alpha > \alpha_{cr} = \varepsilon \), there are two values of \( \Delta k = (k - k_1) \) for which \( \Delta = 0 \).

\[
\Delta k = (k - k_1), \Delta k_{1,2} = \pm 2\sqrt{(\alpha^2 - \varepsilon^2)}
\]

(7)

These conditions are commonly named in the brake squeal literature [14-18] lock-in and lock-out points. Moreover, for values of \( \Delta k \) between the lock-in and lock-out conditions, the imaginary part of one eigenvalue becomes negative, expressing a linear unstable behaviour of the system, while the real part of the two eigenvalues are equal.

Comparing Figure 5 (\( \alpha = 2\varepsilon \)) with the previous Figures 3 and 4, it is possible to notice a dramatic change in the behaviour of the two systems. The interesting feature is that, as the system becomes unstable, the natural frequencies of the system lock together and the two modes become a double mode with a shape that is and remains strongly-coupled.

Moreover, it is interesting to note that \( \text{mac}_{11} = \text{mac}_{12} \) (as well as \( \text{mac}_{22} = \text{mac}_{21} \)) have the same value within the lock-in zone. This is due is due to the fact that, as the lock-in point is reached, both modes become complex and the two eigenvector have only a phase difference,
while the real parts and the absolute values of the two eigenvectors are equal. This strongly-coupled mode shape can be measured as the operating deformed shape during unstable cycles in brake squeal experiments [15, 17].

2.3 Crossing

Among the three cited phenomena, crossing between the eigenvalues is probably the least investigated. Crossing can be obtained by considering a “non-symmetric damped” system, i.e. a system where each degree of freedom has a different value of damping. Let us consider the 2 degrees of freedom system described by equation 8 with structural damping:

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} +
\begin{bmatrix}
k + j\eta + \varepsilon & -\varepsilon \\
-\varepsilon & k_i + j\eta_i + \varepsilon
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

(8)

where \( j \) is the imaginary unit and \( \eta \) and \( \eta_i \) are damping loss factors. In this case, the eigenvalues of the system are:

\[
\lambda_{1,2} = \frac{k + j\eta + k_i + j\eta_i + 2\varepsilon \pm \sqrt{\Delta}}{2}, \Delta = (k + j\eta + k_i + j\eta_i + 2\varepsilon)^2 - 4 \left[ (k + j\eta + \varepsilon)(k_i + j\eta_i + \varepsilon) - \varepsilon^2 \right] =
\]

\[=(k - k_i)^2 + 2j(k - k_i)(\eta - \eta_i) - (\eta - \eta_i)^2 + 4\varepsilon^2\]

When \( k = k_i \), the value of \( \Delta \), depending on the value of \( \Delta\eta = (\eta - \eta_i) \), may become negative, while the system remains stable\(^1\). In particular, the critical value of \( \Delta\eta = \Delta\eta_{cr} = 2\varepsilon \) marks the boundary between veering and crossing behaviour of the system.

Figure 6 shows the simulation for the case \( \eta = 0 \) and \( \eta_i = 3\varepsilon \): it is possible to detect clearly a new situation: the eigenvalues cross each other and the mac functions show that both modes keep their shape while crossing occurs, i.e. \( \text{mac}(k_{11}) \) and \( \text{mac}(k_{12}) \) remain above 0.8.

![Figure 6: Crossing between eigenvalues a) real part of the eigenvalues b) mac functions between modes across the crossing range.](image)

It is interesting to note how \( \Delta\eta \) affects the coupling during crossing. Figure 7 shows the veering behaviour of the two eigenvalues when both \( k_i \) and \( \Delta\eta \) change. In particular, Figure 7b presents the mac functions computed for \( k_i = k_{cr} = 1\text{N/m} \), showing how the transition between veering and crossing behaviour is smooth. In particular, the larger is \( \Delta\eta \) the more both modes keep their shape while crossing.

\(^1\)The system may remains stable even when \( \Delta < 0 \) because there are positive contributions to the imaginary part of the eigenvalues due to the damping loss factors \( \eta - \eta_i \).
3.1 Veering and crossing on higher order systems

In order to highlight veering and crossing phenomena on more higher order system, the case of three coupled beams is considered. The beams have all the same width and are made of the same material. The three beams have all fixed length \( l_1 = 0.3 \text{ m}, \ l_2 = 0.2 \text{ m}, \ l_3 = 0.5 \text{ m} \); the first two beams have fixed thickness \( h_1 = 0.02 \text{ m}, \ h_2 = 0.0005 \text{ m} \) while the third one has thickness variable in the range \( 0.005 < h_3 < 0.06 \text{ m} \).

Figure 8: Sketch of the three flexural beams coupled together

The boundary conditions are represented by hinged ends. The solution is obtained numerically using a Galerkin procedure with a set of 200 modes, determined from the hinged-hinged modes of a beam with constant thickness \( h = h_1 \), and total length \( L = l_1 + l_2 + l_3 \). In Figure 9 the first 12 natural frequencies are plotted versus the varying thickness of the third beam: we can observe that in practice every natural frequency has a veering condition with the following one, excluding the first two modes. This should not wonder: in fact, considering the negligible thickness of the middle beam with respect to the other two, the first two modes are local modes of the middle beam only and do not involve the whole system. In Figure 10 a and b, the MAC between two subsequent modes, before and after the veering, is presented: we can clearly observe that the MAC between the two modes (8-9) is practically zero either at the beginning or at the end, as expected.

Figure 9: Case of weak coupling. Natural frequencies of the system vs. the varying thickness of the third beam and zoom for modes 8 and 9
On the contrary, as observed in Figure 10c, where the MAC is computed between the mode corresponding to the lower natural frequency before veering and the mode corresponding to the upper natural frequency after veering, there is an almost perfect correlation between them, so that it means that, as expected, the two modes interchange.

Figure 10: MAC between modes 8 and 9. a) MAC before veering; b) MAC after veering. c) MAC between the mode corresponding to the lower natural frequency before veering and the mode corresponding to the upper natural frequency after veering.

This result is confirmed by the graph in Figure 11 that shows the mac functions when varying the thickness of the third beam: before veering there is not any correlation between the two considered modes (mac$_{1-2}$ – black, and mac$_{2-1}$ – green) but, after veering, the correlation is almost unit, confirming the interchange.

Figure 11: The mac functions for modes 8 and 9 in the case of veering

It is worthwhile to point out that mac$_{22}$ (before the veering) and mac$_{21}$ (after the veering) decrease so that their value is not anymore one. This is due to the fact that, by varying h$_3$, the second mode changes shape continuously. On the contrary, mac$_{12}$, and obviously, mac$_{11}$ are always close to one because this mode is largely affected by changes in h$_3$ thickness. This is also reflected in Figure 10c, where there is not an unitary mac.
By just adding a structural damping to the third beam ($\eta_3 = 0.05$) without changing any other parameter, a case of crossing can be obtained. In Figures 13 a and b, a crossing between modes 8 and 9 and the corresponding mac functions are presented.

By comparing Figure 11 with Figure 13b, it is possible to notice the radical change in the eigenvalue interaction that, in the latter case, maintain their modal shape while crossing. Similarly to the veering case, it can be noticed that the continuous change in the thickness causes a continuous change in mode 8 shape quantified by a decreasing $\text{mac}_{22}$ function.

3. MODE LOCK-IN, VEERING AND CROSSING IN BRAKE SYSTEMS

This section presents results previously published by the authors in several papers (e.g. [15-18]) showing occurrences of veering, crossing and lock-in in the practical case of brake squeal. These experimental set-ups were designed to allow a control on the natural frequency of the systems so that lock-in conditions could be reached. As a consequence, instability arise and causes an exponential growth of the system response that is mainly characterized by one harmonic coincident with the frequency of the unstable eigenvalue [14,18]. Once the amplitude of the vibrations reaches a certain amplitude, nonlinear effects arise, limiting further increase in the vibration and causing the appearance of super-harmonic of the unstable frequency.

The numerical and experimental results were obtained using different experimental setups that are briefly recalled before presenting the data.

In [18] the beam on disc set-up was used to investigate the effect of “non-symmetric” damping on squeal instability.

![Image of beam on disc setup]

Figure 14: beam on disc setup

The Beam-on-Disc consists of a cantilever beam and a rotating disc pressed against each other using a dead weight. Figure 14 shows a photograph and a scheme of the set-up: the disc and the cantilever beam are both made of steel.

Using this set-up, an experimental parametric analysis was conducted by adding small masses to the beam, allowing to change the natural frequencies of the beam with the aim of measuring the lock-in plot. The results are presented in Figure 15 for the coupling between the second mode of the beam and the (0,4)$^2$ mode of the rotor. The experiment is conducted by adding step by step mass to the beam and, at each step, measuring the natural frequencies of the system (Figure 15b). Reached a certain distance between the natural frequency of the beam and that of the disc the system becomes unstable and squeal starts. During squeal, only

\[2\]

The disc modes are denoted as $(n,m)$ where $n$ is the number of nodal circumference, and $m$ is the number of nodal diameters. Because of the axial symmetry of the disc, all the modes are double modes. The contact between disc and pad brakes the symmetry of the system causing the two double disc mode to split; in this case, $(n,m+)$ mode denote the one that has an antinode at the pad position that is usually the higher frequency one.
one frequency can be measured: the squeal frequency (Figure 15b) moreover, it is possible to measure with a laser scanner vibrometer the unstable deformed shape that is presented in Figures 15 d and e, confirming the fact that, in the instable range, a strongly-coupled modal shape is present, while before and after lock-in, two different natural frequencies, each associated with the mode of either the beam or the disc, are measured.

Figure 15. Experimental lock-in plot in the beam on disc set-up and squeal deformed shapes

The tribobrake [16] (Figure 16a) is an experimental rig designed to study the tribological aspect of the squeal in brakes. The geometry of the system is characterized by a steel rotor while the brake pad consists of a cube 10x10x10mm pressed against the rotor by means of dead weights.

Figure 16: The tribobrake. a) experimental set-up; b) complex eigenvalue analysis performed on a predictive model of the setup

The FE model of this set-up is characterized by a non-symmetric stiffness matrix, the asymmetric terms being due to the presence of the friction force between the disc and the pad:

\[
\begin{align*}
F_x^d &= k_n \cdot (z^d - z^p) \\
F_y^d &= -\mu \cdot k_n \cdot (z^d - z^p) \\
F_x^p &= -k_n \cdot (z^d - z^p) \\
F_y^p &= \mu \cdot k_n \cdot (z^d - z^p)
\end{align*}
\]  

(9)

where the superscripts d and p stand for disc and pad respectively, \( k_n \) is the contact normal stiffness between the nodes in contact, \( z \) the displacement in the direction normal to the contact surface, \( F_x \) and \( F_y \) the contact forces in the normal and tangential directions, respectively and \( \mu \) the friction coefficient. This model is able to detect the squeal instabilities occurred during experiments.

In Figure 16b, the numerical results obtained from the complex eigenvalue analysis of the FE model are presented. The results show how the interaction between the eigenvalues
become more complex than in the case of the two-DOFs system but, nevertheless, veering and lock-in still appear.

In particular, two unstable zones are detected (unstable eigenvalue are plotted as large black dots): one in the 7600Hz range, where the mode of the pad interacts with the (1,0) mode of the disc through a typical lock-in. In the frequency range around 7700Hz a three mode interaction is present involving lock-in between two double modes of the disc (0.6), and a veering between the pad mode and the unstable double mode (0,6) of the rotor.

Another experimental rig for squeal study is the laboratory brake [15] (Figure 17) that consists of a stainless steel disc while the calliper is made by two steel beams (A and B) that hold the brake pads (C and D). A reduced order model of this system was developed starting from measured modes of the disc and the beam when not in contact, and coupled to a theoretical one-DOF model of the pad. The reduced model is characterized by a non-symmetric stiffness matrix whose non-symmetric terms are due to the friction force. A non-symmetric damping is also considered in the model in that the damping of the two pads is one order of magnitude higher than the damping of the disc and the beam.

Figure 18 shows an example of instability detected by the reduced model: in the frequency range around 6500Hz squeal events, involving the (0,6+) mode of the disc, are measured. The model presents in this case an unstable crossing of the eigenvalues. In particular, being the difference among pad and disc damping larger than the critical value, an eigenvalue crossing is possible. In this case, however, the presence of the gyroscopic term allows instability to occur (black zone in Figure 19).

Figure 17: The laboratory brake  a) experimental set-up; b)Schematic top view

4. CONCLUSIONS

In either simple or high-order coupled systems, two eigenvalues interact when are “close” to each other they. Three main type of interactions can occur: veering, lock-in or crossing and consequently system response is generally largely affected by the type of the interaction.

Among the different results shown in this paper, it is worthwhile to point out that the damping, when not uniformly distributed among the degrees of freedom of the system, affects the eigenvalue interaction in a radical way. In particular, contrary to the general feeling, while
a high damping merges two close peaks in the FRF, it may induce a transition from veering to crossing, allowing the two modes to keep their individuality.

Similarly, if the undamped system has gyroscopic terms and exhibits lock-in instability, by adding a non uniform damping, the eigenvalue interaction is altered and a wide range of unstable behaviors can emerge.

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