AN OPTIMIZED IDENTIFICATION METHOD
FOR MODULAR MODELS OF RUBBER BUSHINGS

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ABSTRACT

Rubber bushings are important for automotive manufacturers, for ensuring the vehicle vibration comfort (in terms of vibrations and noise). Ideally the bushing design can be optimized based on virtual simulation models; after all, the more design decisions are taken in an early stage, the lower the design costs (as the physical prototype validation phase can be shortened) and the better the product quality (as earlier decision-making implies still a larger range of feasible design modifications based on the virtual simulations). For this purpose, including rubber bushing behaviour in multibody vehicle dynamic simulations is a crucial task, comprising the solution of two sub-problems: the mathematical modelling to define a constitutive mathematical force-displacement relationship to reproduce the bushing behaviour, and the identification procedure to fit the selected model to the experimental data.

Modular modelling was recently presented as an efficient approach, resulting in a good trade-off between the complexity of the mechanical characteristics of bushing components, and the computational costs required to incorporate bushing component models with sufficient fidelity in the CAE simulation. On the other hand, the state-of-the-art identification procedures can be classified into two main groups: parameterization approaches, requiring an excessive amount of user interaction and expertise, and optimization approaches, which may require a high number of deterministic calculations and involves the risk to not converge to the optimal design.

This paper presents an innovative fast and robust identification tool that relies on efficiently combining a parameterization technique with a nonlinear fitting optimization algorithm. It is shown that the results fit very well experimental data reported in recent literature. Moreo-
ver, the method compares favorably to other recent prediction methods in terms of computational efficiency, yielding the same (or better) accuracy than prior art.

1. INTRODUCTION

Rubber bushings are mainly used in vehicle design to connect moving parts to each other in order to minimize vibration, wear, and transmission of noise, or as passive suspension systems to control unwanted vibrations induced by road roughness and engine spin. In their essential configuration, they generally consist of rubber tubes bonded on their outer and inner surfaces to rigid metal layers.

Including rubber bushing in multibody vehicle dynamic simulations has thus become a crucial task in order to allow the assessment and optimization of vehicle performance from the early stage of the design process, or complement and partially replace physical testing [1]. At present, bushing simulation is a priority R&D topic in the state of the art, as rubber materials modeling and characterization methodologies are still evolving. Many automotive OEMs perform in-house R&D in this topic area, because of the strategic importance of vehicle NVH and comfort, which are key for the brand perception of their cars.

Typical rubber properties, such as hyper-elasticity, viscoelasticity and static friction, affect the bushing component behavior which exhibits a complex mixture of different features like nonlinearity, frequency and amplitude dependence, hysteresis [2-4]. Therefore, each specific behavior must be modeled in order to achieve a reliable prediction [5,6] of the whole bushing characteristics.

Although several simple models have been used in the past to model viscoelasticity, i.e. frequency dependence, such as the Kelvin–Voigt cell (spring and damper in parallel), the Maxwell cell (spring and damper in series), the Zener cell (spring and Maxwell cell in parallel), better description is obtained by a summation of Maxwell cells [7-9] or by adopting fractional derivative models [10-12]. In the first case the number of model parameters is higher leading to a more challenging identification, but the resultant time-domain model is easier to implement; in the second case the number of parameters to estimate is only two, but the time-domain numerical implementation is relatively more computationally expensive [13].

Friction behavior, i.e. amplitude dependence, which is significant for rubber components filled with carbon black [14-17], is modeled by adding a friction cell in parallel to the elastic and viscous cells. As the basic stick-slip element, e.g. the linear Coulomb friction model, exhibits discontinuous behavior, several attempts to reproduce a smooth functional relationship have been proposed [18-21].

In order to meet the different and case-dependent requirements, a modular and generally applicable phenomenological model for rubber bushings has been recently developed at Daimler, which is used as a uniaxial displacement-force element in simulation environments [22]. Due to the fact that the dependence on problem-specific requirements and scope of application can make specific models still exhibit evident deficiencies in accuracy and applicability, such adaptable modeling is proposed as a valuable approach to achieve more user-friendly interaction, especially when model parameters have to be estimated from experimental data or, if not available, from experience-based key characteristics. For this purpose, in [22], an effective parameterization is also adopted to directly identify parameters corresponding to specific bushing models. Similar approach have also been used in [19], and in [10, 23] for the frequency independent parameters. Alternatively, optimization algorithms may be used for parameters estimate, which may differ on the appropriate selection of the objective function and weighing factors. Such approach has been used, for instance, in [12, 24] and in [10, 23] for the frequency independent parameters. Graphical methods have been also used for the estimation of parameters of a pure viscoelastic model [25], and very recently, a method has been proposed, where identification has been achieved in the time domain [26].

The aim of this paper is to extend the work done in the above-mentioned references, and in particular in [22], by adopting a similar modular modeling for rubber bushing, while propos-
ing a fast and robust identification tool that relies on efficiently combining a parameterization technique with a nonlinear fitting optimization algorithm. Such adaptable approach in the identification step as well, may effectively cope with the specific expertise of the user and with the trade-off between fast or detailed results, which are often conflicting requirements.

2. MODULAR MODEL

Adopting the modular strategy suggested by [22], two different variants are presented along the paper: the Maxwell-Cells-based model (MCM), and the Spring-pot-based model (SPM).

Figure 1. Modular architecture of the two variants proposed along the paper: the Maxwell-Cells-based model (up) and the Spring-pot-based model (down)

As depicted in Figure 1, the two proposed models share a common part that includes a progressive spring element, a friction element, and a linear damper; whereas the differences regard only the use of the Maxwell cells elements instead of the spring-pot model, respectively for the MCM and the SPM.

Note that all the modular elements are combined in parallel: this permits to compute the global force of the model as the summation of the forces given by each element, while the input displacement is the same for each element.

2.1 Nonlinear spring

The complete model for the nonlinear spring element relies on a lookup table, which is defined using a finite set of force-displacement experimental points. Therefore, the reaction force, $F_s$, is computed as a function of the current relative displacement, $X$, of the spring (i.e. $X = 0$ correspond to the unstretched position).

$$F_s = \text{LUT}(X)$$

2.2 Static Friction element

The mathematical formulation of the friction element was elaborated using the friction force formulation proposed by Berg in [19]. The friction force, $F_f$, at a given simulation points de-
pends not only on the displacement, \( x \), but also on a reference state \((x_s, F_{fs})\), that is defined respectively by the force and the displacement values corresponding to the last turning point of the current hysteresis branch.

The full computation of the friction element, in a time-domain simulator should involve two main steps:

- updating state variables, (default values could be set to 0, at the beginning of the simulation)

\[
(x_s, F_{fs}) = \begin{cases} 
(x, F_f), & [(x > x_s) \land (\dot{x} < 0)] \lor [(x < x_s) \land (\dot{x} > 0)] \\
(x_s, F_{fs}), & \text{otherwise}
\end{cases}
\]  

- computing current friction force, \( F_f \),

\[
a = \frac{F_{fs}}{F_{max}}
\]

\[
F_f = \begin{cases} 
\frac{F_{fs}}{x_f (1 + a)} - (x - x_s) (F_{max} + F_{fs}), & x < x_s; \\
\frac{F_{fs}}{x_f (1 - a) + (x - x_s)} (F_{max} - F_{fs}), & x > x_s; \\
\end{cases}
\]  

According to the given equation, the friction behavior is modeled exhaustively as a function of two parameters, \( x_f \) and \( F_{max} \). As reported in [19], the steady-state hysteretic loop introduced by the same model is characterized by a force amplitude, \( F_{f0} \), given by

\[
F_{f0} = \frac{F_{max}}{2x_f} \left( x_f^2 + x_0^2 + 6x_f x_0 - x_f - x_0 \right),
\]  

and the corresponding energy loss per cycle, \( E_f \), is given by

\[
E_f = 2F_{max} \left( 2x_0 - x_f (1 + a_0)^2 \ln \frac{x_f (1 + a_0) + 2x_0}{x_f (1 + a_0)} \right), \quad \text{where: } a_0 = \frac{F_{f0}}{F_{max}}
\]  

2.3 **Maxwell-Cell based dynamic behavior**

Following the suggestions reported in [22], the Maxwell-cells elements were coupled with a linear damper element, in order to compensate possible phase decay behaviour. Thus, the resulting complete model formulation could be expressed in the frequency domain, in term of the corresponding complex stiffness, according to the following expression:

\[
G(s) = s \ast \left( d + \sum_{i=1}^{n} \frac{c_i k_i}{s c_i + k_i} \right)
\]

where \( d \), is the linear damping coefficient, \( k_i \), and \( c_i \), are respectively the stiffness and the damping coefficients of the \( i^{th} \) Maxwell-cell, and \( n \), is the total number of Maxwell cells.

2.4 **Spring-pot based dynamic behavior**

The spring-pot element mathematical background is based on the idea to apply fractional derivatives in order to have a sound simulation of the visco-elastic behaviour on a wide frequency range, without suffering the phase-decay problems of the Maxwell cell implementation. The exact time-dependent mathematical model is then expressed as:

\[
F_{sp}(t) = b D^\alpha x(t), \quad \alpha, b \in \mathbb{R}, \quad 0 < \alpha < 1, \quad b > 0.
\]  

The spring-pot force, \( F_{sp}(t) \), is obtained as the not-integer differentiation of the input displacement, \( D^\alpha x(t) \). The model is completely defined through the definition of two parame-
ters: the gain factor, \( b \), constrained to assume a real positive value; and the differentiation order, \( \alpha \). It is worth noting that the latter parameter is constrained \( (0 < \alpha < 1) \) to assume a real value ranging between \( \alpha = 0 \), corresponding to the pure linear spring equivalent case, and \( \alpha = 1 \), corresponding to a viscous damper.

However, the inclusion of this model into an integration scheme could not be realized without introducing some level of approximation. Following the suggestion reported in [22], we used the following approximation, which relies on the discrete Grünwald definition and the computation of the Gamma function, \( \Gamma(*) \):

\[
F_{sp} \approx \sum_{m=0}^{n-1} A_m X_{n-m}, \quad \text{where: } A_m = b \Delta t^{-\alpha} \frac{\Gamma(m - \alpha)}{\Gamma(-\alpha)\Gamma(m + 1)}.
\]

It is worth noting that for saving the computation efforts due to the Gamma function evaluation, the same approximation could be also expressed in a more compact form,

\[
F_{sp} \approx \overline{A}_m \cdot \overline{X}_h
\]

as the scalar product between the coefficient vector, \( \overline{A}_m \), and the history vector, \( \overline{X}_h \), which stores the last \( n \) values of the input displacement, sampled at a given constant rate interval, \( \Delta t \). The length of these vectors, \( n \), as well as the integration time step, \( \Delta t \), can be considered as tuning parameters related to the desired level approximation. An interesting analysis about their impact on the quality of the approximation may be found in [22].

The frequency-domain representation, corresponding to the proposed approximated computations, coupled with a linear damper element, is given by:

\[
G(j\omega) = j\omega d + \sum_{m=0}^{n-1} A_m (\cos(\omega m \Delta t) - j \sin(\omega m \Delta t))
\] (8)

3. FAST AND ROBUST IDENTIFICATION

The identification methodology provided in this section aims to compute the set of parameters for any one of the proposed models (MCM or SPM), using as input the static and dynamic characteristic which are commonly used to analyse the bushing components behaviour.

Generally speaking, an identification problem could be easily redefined as multi-variable fitting problem, for which a proper optimization routine can be used for computing the optimal set of parameters. However, in presence of the computational complexities of the models proposed in this paper, the use of a common fitting strategy may result in unpredictable time execution and/or it may lead to the inability to converge to an optimal solution. Sedlacek et al. [22] originally proposed the parameterization approach as a good alternative to the optimization methodology, able to give an excellent level of robustness.

On the other side, this paper exploits the parameterization approach with the intent to counterbalance the above mentioned typical challenges of an optimized identification process. This is achieved with a mixed approach which relies on the parameterization to successfully initialize the starting value for the critical parameters. In facts, using the parameterized solution as the starting guess, the fitting is able to reach an optimal solution in a few iterations.

The overall identification tool can be divided in five main tasks, whose details are given in the following sections:

a) input preprocessing tasks;
b) friction module parameterization;
c) progressive spring parameterization;
d) initialization of dynamic branch;
e) nonlinear fitting of the whole model.
3.1 Input dataset processing

Similarly to the work presented in [1], the proposed algorithm elaborates simultaneously both the quasi-static input curve and the dynamic curves. In particular, the experimental curves are processed according to the following steps:

1. Static curve preprocessing tasks:
   1.a First, the quasi-static curve input, is analyzed assuming that the quasi-static load was carried out with a symmetric range, i.e. with harmonic displacements ranging in the \([-x_s, x_s]\) interval.
   1.b Then, two auxiliary sets \(F_{su}\) and \(F_{sl}\) are defined splitting the static curve between the upper-cycle force values (green curve in Figure 2 - left), and the lower-cycle force values (red curve).
   1.c The energy loss per cycle of each hysteretic cycle, \(L_s\), is estimated evaluating the extent of the area by means of numeric integration.

2. Dynamic curves processing tasks:
   2.a An arbitrary frequency \(\omega^*\) is selected, as the average value of the frequency range of the whole dataset;
   2.b then, the difference, \(\Delta M\), between the lower amplitude magnitude curve and the larger amplitude one is computed, interpolating the dynamic curves at the \(\omega^*\) frequency points.

3.2 Friction parameterization

In this task, a system of two nonlinear equations is solved in order to evaluate analytically the friction parameters. First equation is deduced considering that for any given frequency, \(\omega^*\), the overall magnitude (i.e. dynamic stiffness) component of the bushing behavior can be decomposed in a static part, \(M_s\), which does not vary with frequencies but it is dependent on the amplitude of the input; and a dynamic part, \(M_{d,\omega^*}\), that varies only as a function of the frequency, but is the same for any amplitude of excitation. According to the mentioned assumptions, \(\Delta M\) can be expressed as:

\[
\Delta M = M_{1s} + M_{1d,\omega^*} - M_{2s} - M_{2d,\omega^*} = M_{1s} - M_{2s}
\]

Considering that the static contribution to the magnitude curve is given by the composition of the friction element and the progressive spring element, the generic static component of the complex stiffness is given by:

\[
M_s = \frac{F_s + F_f}{x_a},
\]

where \(x_a\) is the given amplitude and \(F_s\) and \(F_f\) forces are computed respectively using equations (1) and (3). Thus, the magnitude difference, \(\Delta M\), can be decomposed as:

\[
\Delta M = \left(\frac{F_{s1}}{x_{a1}} - \frac{F_{s2}}{x_{a2}}\right) + \left(\frac{F_{f1}}{x_{a1}} - \frac{F_{f2}}{x_{a2}}\right) = \Delta M_{spring} + \Delta M_{friction}
\]
Ignoring the nonlinearity effects, the term $\Delta M_{spring}$ is set to zero; and after applying twice equation (4), $\Delta M$ can be expressed as a function of the friction parameters, yielding

$$
\Delta M \equiv \Delta M_{friction} = \left( \frac{F_{f,1}}{x_{a,1}} - \frac{F_{f,2}}{x_{a,2}} \right) = \frac{F_{\text{max}}}{2x_f} \left[ \frac{1}{x_{a,1}} \left( \sqrt{x_f^2 + x_{a,1}^2 + 6x_f x_{a,1} - x_f - x_{a,1}} \right) - \frac{1}{x_{a,2}} \left( \sqrt{x_f^2 + x_{a,2}^2 + 6x_f x_{a,2} - x_f - x_{a,2}} \right) \right],
$$

that can be formulated as

$$
F_{\text{max}} = \frac{2 * \Delta M_{friction} * x_f}{\sqrt{x_f^2 + x_{a,1}^2 + 6x_f x_{a,1} - x_f} - \sqrt{x_f^2 + x_{a,2}^2 + 6x_f x_{a,2} - x_f}} \quad (10)
$$

where $F_{\text{max}}$ is expressed as a function of the other friction parameter, $x_f$.

The second equation relates the loss energy per cycle of the quasi-static curve to the friction element. In particular, the following is obtained applying equation (5) for the quasi-static loop:

$$
L_s = 2F_{\text{max}} \left( 2x_s - x_f(1 + a_s)^2 \ln \frac{x_f(1 + a_s) + 2x_s}{x_f(1 + a_s)} \right),
$$

with:

$$
a_s = \frac{\sqrt{x_f^2 + x_s^2 + 6x_f x_s - x_f - x_s}}{2x_f} \quad (11)
$$

Finally, the nonlinear equation obtained by substituting (10) into (11) is solved numerically for $x_f$. Consequently, $F_{\text{max}}$ is estimated using (10).

### 3.3 Progressive Spring parameterization

Once the friction parameters, $F_{\text{max}}$ and $x_f$, are estimated the progressive stiffness characteristic of the spring element can be computed, according to the following:

$$
F_{nls,u} = F_{su} - F_{fu} \quad \text{and} \quad F_{nls,l} = F_{sl} - F_{fl} \quad \Rightarrow \quad F_{nls} = \frac{F_{nls,u} + F_{nls,l}}{2}
$$

The computation relies on removing the friction contributions, $F_{fu}$ and $F_{fl}$, from the quasi-static input curve parts, $F_{su}$ and $F_{sl}$, that were previously computed. Then, $F_{fu}$ and $F_{fl}$, are estimated applying the time domain model described by equations (2) and (3), using as input displacement, same points stored into the two portion of the experimental static curve, previously elaborated as $F_{su}$ and $F_{sl}$. Finally, the LUT describing the progressive spring behavior is obtained by averaging $F_{nls,u}$ and $F_{nls,l}$, and then pairing the resultant force vector, $F_{nls}$, with the corresponding input displacement vector.

### 3.4 Initialization of dynamic branch

According to the modular architectures introduced in the previous section, dynamic behavior is modeled using a combination of Maxwell cells (MCM variant) or a single spring-pot model element (SPM variant). This section describes the heuristics used to initialize the parameters corresponding to the dynamic branch, with a set of feasible values which gives the final optimization step the possibility to reach a robust solution.

For the MCM variant, given the integer number of cell to be used, $m_c$, and considering the frequency range limited between, $f_l$ and $f_h$, the Maxwell cell parameters are computed according to the followings:

$$
\forall i \in \mathbb{N}, i = 1 \ldots mc; \Rightarrow k_i = 1; \quad \tau_i = \frac{1}{2} \left( 10^{f_l + (f_h - f_l) \frac{i - 1}{mc}} + 10^{f_l + (f_h - f_l) \frac{i}{mc}} \right); \quad c_i = \frac{k_i}{\tau_i}
$$

For the SPM variant, the parameters of the spring-pot are initialized as: $a = 0.5, b = 0.5$
The damping parameter is initialized to zero, for both variants.

3.5 Nonlinear fitting of the whole model

The final optimization step of the identification strategy is formulated as a nonlinear minimization problem, solved using the Levenberg-Marquardt algorithm [27, 28]. To this end, the objective functional is defined as the sum of the squared distances between each simulated point and its corresponding measure along the dynamic input curves.

In particular, for each experimental point of the dynamic curves, the procedure extract the following quantities: \( f_i \) and \( X_i \), which are, respectively, frequency and amplitude of the exciting input displacement; \( M_i \) and \( P_i \), which are, respectively, the magnitude and loss angle of the dynamic stiffness of the bushing. Then, the overall computation of the functional is achieved by repeating the following steps, for each experimental point:

- given the friction parameters, parameterization of the LUT for modeling the progressive spring follows an approach similar to that described in Section 3.3. This is crucial for maintaining robustness and convergence of the algorithm under control;
- the amplitude of the friction force, \( F_{fi} \), and the corresponding energy loss, \( E_{fi} \), are computed using (4) and (5);
- the spring element force is estimated by LUT evaluation: \( F_s = LUT(X_i) \);
- the stiffness due to static and friction components is computed as: \( K_s = \frac{F_{fi}+F_s}{X_i} \).
- the complex modulus, \( G_{dyn} \), due to the dynamic components is evaluated according to the proper model (using (6) for MCM and (8) for SPM):
  \[
  G_{dyn} = \text{model}(P, 2\pi f_i).
  \]
- the magnitude of the global complex modulus is obtained as
  \[
  M_{ci} = \| K_s + G_{dyn} \| = \sqrt{(K_s + Re(G_{dyn}))^2 + Im(G_{dyn})^2}
  \]
- the global loss angle is derived as:
  \[
  P_{ci} = \sin^{-1}\left(\frac{E_{fi}^2 + K_{dyn} \sin P_{h_{dyn}}}{M_{ci}}\right)
  \]
- finally, the residual error of each experimental point is evaluated as:
  \[
  E_i = \lambda \left(\frac{M_{ci} - M_i}{M_i}\right)^2 + (1 - \lambda) \left(\frac{P_{ci} - P_i}{P_i}\right)^2,
  \]
  where the \( \lambda \) is a weighting parameter, that the user could adjust interactively in order to balance between the pure fitting of the magnitude curves (\( \lambda = 1 \)), and the pure fitting of the phase curves (\( \lambda = 0 \)). In the final version of the tool, this parameter is adjusted interactively moving a slider of the graphic interface.

The functional, corresponding to the current set of parameters is obtained by summing all the residuals, \( E_i \), obtained for each experimental point. Then, according to the Levenberg-Marquardt algorithm, the objective function is evaluated iteratively in order to update the estimation of the optimization variables until convergence is reached.

4. RESULTS

A validation of the proposed tool has been achieved testing the proposed models and the corresponding identification strategies, using the experimental data reported in [22].

Figure 3 depicts the fitting performance of the two variants proposed for the static characteristic of the bushing. The simulated static characteristic were obtained using the whole modular model, with a quasi-static (0.01 Hz) load having the same amplitude of the experimental static characteristic. As illustrated, the fitting of the static characteristic fits very well
the given experimental curves, for both models, without showing particular differences among
the two proposed variants.

Figure 3. Measured and simulated static characteristic of the bushing model variants: MCM
(left) and SPM (right)

The fitting of the dynamic behavior presents interesting aspects that are shown from Figure
4 to Figure 7. Figure 4 and Figure 5 present, respectively, the performance of the SPM and the
MCM variants, obtained with a weighting factor ($\lambda = 0.6$) chosen with the intent of com-
promising between the fitting of the magnitude curves and the loss angle ones.

Figure 4. SPM performance in comparison with measured dynamic characteristics ($\lambda = 0.6$)

As expected, increasing the weight of the magnitude curves ($\lambda = 0.9$) produces better re-
results in term of fitting the dynamic stiffness, but the quality of the loss angle is reduced
(Figure 6 and Figure 7).

Figure 5. MCM performance in comparison with measured characteristics ($\lambda = 0.6$)
Comparing with the results proposed in [22], the identification methodology proposed in this paper express the ability to match the dynamic stiffness curves with the same level of quality, for both the variant without any particular influence from the typology of the model used. At the same time, the proposed approach gives a better fitting on the loss angle curves especially for the MCM variant.

Using a commodity notebook PC (Dual processor @2.53 GHz, 4.00 GB RAM), the execution of the whole identification process elapsed in all cases only a few seconds. This permits to include the process in a GUI, and to provide immediate results to the user.

![Figure 6. MCM performance in comparison with measured dynamic characteristics (λ = 0.9)](image1)

![Figure 7. SPM performance in comparison with measured dynamic characteristics (λ = 0.9)](image2)

5. CONCLUSIONS

Recently, modular modelling has been presented as an efficient approach, resulting in a good trade-off between the complexity of the mechanical characteristics of bushing components, and the computational costs required to incorporate bushing component models with sufficient fidelity in the CAE simulation. This paper presented an original identification approach able to combine the quality of results obtained with recently proposed parameterization methods, with the increased level of automation typical of the nonlinear fitting approaches.

Comparing to state of the art and available industrial parameterization approaches, the newly proposed methodology offers a great flexibility without requiring specific experience. Moreover, thanks to the efficiency of the proposed methodology, the user has the possibility to evaluate many model variants in the design timeframe, and test interactively his/her decisions. For instance, this offers the design engineer the opportunity to find the right balance between stiffness and loss angle fitting. This has been demonstrated in this paper, in which it
is shown that the presented identification process is able to produce a high-quality match of both stiffness and loss angle curve.

One limitation of the current implementation is the assumption that the quasi-static load curve has a symmetric shape. Moreover, the model with the spring-pot element requires that a fixed-step integration scheme is used during simulations, which can be a penalty factor for the computational efficiency of the full system model. More in general, the 1D nature of the proposed models does not allow for taking into account cross-coupling effects between different degrees of freedom. Further investigations are foreseen to analyze whether and to what extent the abovementioned limitations can be overcome.

Finally, exploiting the modular nature of the proposed mixed approach, the current version of the identification process could be easily adjusted to reflect any kind of change (the revision of existing elements and/or the inclusion of new ones) made to the modeling architecture in order to produce a better fit of the dynamic behavior of the rubber bushings, or even other similar components.

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