COUPLED REDUCED ORDER MODEL-BASED STRUCTURAL-THERMAL PREDICTION OF HYPERSONIC PANEL RESPONSE

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ABSTRACT

This paper addresses some aspects of the development of a fully coupled thermal-structural reduced order modeling of planned hypersonic vehicles, most notably the construction of the thermal and structural bases. A general framework for this construction is presented and demonstrated on a representative panel considered in prior investigations. The thermal reduced order model is first developed using basis functions derived from appropriate conduction eigenvalue problems. This basis is validated using published data of which it is found to provide an accurate representation. The coupling of this thermal model with a recently developed nonlinear structural reduced order model of the same panel is next considered. This coupling requires first the enrichment of the structural basis to model the elastic deformations that may be produced consistently with the thermal reduced order model. This step is detailed for the present panel and then the temperature dependent coefficients of the structural model are determined. The validation of the combined structural-thermal reduced order model is carried out by comparison with full finite element results (Nastran here) corresponding to pure mechanical loads, pure thermal loads, and combined mechanical-thermal excitations. Such comparisons are performed here on static solutions with temperature increases up to 2700R and pressures up to 3 psi for which the maximum displacements are of the order of 3 thicknesses. The reduced order model predicted results agree well with the full order finite element predictions in all of these various cases.
1. INTRODUCTION

Proposed future reusable hypersonic air vehicles will be subjected to extreme environments of high thermal, aerodynamic, and acoustic loading and thus their structural response will be large enough to exhibit significant geometric nonlinearity. Additionally, the analysis requires a multidisciplinary framework for the coupling of aerodynamics, heat transfer, and structural dynamics [1-5]. Traditional approaches based on full order computational models are expected to be too computationally demanding for full scale problems with structural nonlinearity and the multidisciplinary coupling, especially considering the duration of analysis that is needed for an accurate prediction of the accumulated damage in the structure.

Recently, within the last decade or so, reduced order models have emerged as a potential solution to this problem, e.g. see [5] and the review of [6]. It has in particular been shown that the coupling between the heat transfer and structural dynamics problems can be modeled in a straightforward fashion [7-11]. The effective number of degrees of freedom in the structural-thermal problem can then be drastically reduced, producing a dramatic increase in the speed of analysis.

The present investigation is part of a long term effort to confirm the application of reduced order models to this multidisciplinary problem and assess their computational benefits. This effort focuses more specifically on the representative panel of Fig. 1 studied in particular in [4] using coupled full order methods. In fact, that investigation revealed clearly the importance of fully coupling the three disciplines previously mentioned, obtaining notably different results in fully coupled vs. one-way coupled analyses. The aim of this current project is thus to accurately model the structural response of the panel of Fig. 1 and its temperature distribution using reduced order models which can be marched in time in flight conditions similar to those described in [4] and including the variations of properties with temperature as well as the coupling with the aerodynamics.

A first effort in this direction, i.e. [10], has focused on the development of an isothermal structural model of the panel which included validation under static and dynamic uniform pressure loadings. As second step, this work is extended here to a coupled structural-thermal reduced order model that is capable of (i) closely representing the temperature fields of the one-way coupled analysis of [4] and (ii) predicting the structural response to uniform pressure loads as well as the temperature distribution. Such a reduced order modeling capability has already been successfully achieved in [11] in a different format. Comparisons of predictions obtained by the present reduced order model with full finite element computations with Nastran demonstrate the appropriateness of the coupled reduced order model. The extension of the current effort to include the time marching of the temperature field with temperature dependent properties and the inclusion of aerodynamics and its comparison with the results of [4] will be presented in a future report.

Figure 1. Representative hypersonic panel
2. THERMAL-STRUCTURAL ROM FORMULATION

2.1. Expansion Representations and Basis Functions

In the structural and thermal reduced order models (ROMs) considered here, the temperature and displacements of the finite element nodes, stacked in the time varying vectors \( \mathbf{T}(t) \) and \( \mathbf{u}(t) \), are expressed in modal expansion forms, i.e.

\[
\mathbf{T}(t) = \sum_{n=1}^{N} \tau_n(t) \mathbf{T}^{(n)}
\]

\[
\mathbf{u}(t) = \sum_{n=1}^{M} q_n(t) \mathbf{u}^{(n)}.
\]  

In these equations, \( \mathbf{T}^{(n)} \) and \( \mathbf{u}^{(n)} \) are the thermal and structural basis functions, or modes, while \( \tau_n(t) \) and \( q_n(t) \) are the time-dependent thermal and structural generalized coordinates.

The selection of the bases \( \mathbf{T}^{(n)} \) and \( \mathbf{u}^{(n)} \) represents the first and probably most significant challenge in the construction of thermal and structural reduced order models; if the structural response and temperature fields are not well represented within these bases, the predictions of the reduced order model will in general be poor. Key is thus to develop a basis construction technique that relies on the least prior information about the displacement and temperature fields. Below is a general description of the currently proposed framework, its specific application to the panel of Fig. 1 is detailed below (section 4).

Consider first the construction of the structural basis. Following standard practice from linear structural dynamics, one selects first the set of linear modes which are in the excitation band (step 1). Other modes are also anticipated to be needed to capture the effects of the geometrical nonlinearity. It was argued in [12] to extract them from nonlinear static responses of the structure subjected to loadings that induce, in linear analyses, the response to be along the basis of step 1. The modes constructed in this step 2 are referred to as “dual modes” and are thus associated with the linear modes of the structure. The combination of linear and dual modes has been seen to be particularly efficient as demonstrated in the validations of [6,12] and the recent application to a complex structure [13]. In the present context of thin walled structures, the modes of step 1 are primarily transverse for “low” frequency excitations while those of step 2 have significant or dominant in-plane components and are thus notably stiffer.

In structural only situations, the above steps 1 and 2 would be sufficient. However, when a nonzero temperature field is applied, associated deformations of the structure take place by way of thermal expansion. In the present thin walled structures, these deformations are mostly in-plane with the possibility of buckling-induced transverse displacements. The latter motions, when they take place, are typically well represented by the linear modes of step 1 and thus have not been found to necessitate an increase of the basis. This is however not the case of the in-plane expansion which is usually not well represented by the dual modes the purpose of which was to capture the effects of structural nonlinearity. Thus, it is expected that the structural basis will need to be expanded in structural-thermal problems. In this regard, note that the in-plane thermal expansion to be captured is not expected to produce significant in-plane nonlinearity. Thus, the basis enrichment proposed for thermal-structural coupling problems will be based on the static linear structural responses induced by each thermal mode in turn. To minimize the size of the structural basis, a proper orthogonal decomposition (POD) will be carried out and the dominant eigenvectors will be selected as step 3 enrichments of the structural basis. Note that when the panel exhibits temperature dependent
structural properties, the static linear responses should be determined for varying magnitudes of the temperature distributions. 

A similar framework is proposed here for the construction of the thermal basis. That is, a first basis will be adopted that is based on the eigenvectors of the heat conduction problem alone at a particular temperature (as in step 1 above). If the thermal properties of the structure are temperature dependent, the eigenvalue problem should be repeated under varying magnitudes of the temperature distributions to obtain, in a POD format, a first enrichment which is similar to step 2 above. That basis will be enriched/modified to account for the expected heat convection from the aerodynamics, i.e. to account for the multidisciplinary interaction as in step 3 above. A similar effort could also be carried out to account for the structural effects, i.e. latency and change of geometry, but is not expected to be necessary given the small magnitude of these effects.

2.2. Governing Equations and ROM Parameter Identification

Having established the basis for the representation of the temperature and displacement fields, the next task is the derivation of the governing equations for the generalized coordinates \(\tau_n(t)\) and \(q_n(t)\) which can be obtained from finite deformations thermoelasticity. Assuming that the material properties (elasticity tensor, coefficient of thermal expansion) do not vary with temperature, it is found, e.g. [6-10], for the structural generalized coordinates that (summation over repeated indices assumed)

\[
M_{ij} \ddot{q}_j + D_{ij} \dot{q}_j + \left[K_{ij}^{(1)} - K_{ij}^{(th)} \tau_l\right] q_j + K_{ij}^{(2)} q_j q_l + K_{ijl}^{(3)} q_j q_l q_p = F_i + F_i^{(th)} \tau_l . \tag{3}
\]

In this equation, \(M_{ij}\) denotes the elements of the mass matrix, \(K_{ij}^{(1)}, K_{ijl}^{(2)}, K_{ijlp}^{(3)}\) are linear, quadratic, and cubic stiffness coefficients and \(F_i\) are the modal mechanical forces. The parameters \(K_{ijl}^{(th)}\) and \(F_i^{(th)}\) represent the sole coupling terms with the temperature field, see [6-10]. When variations with temperature of the elasticity tensor and coefficient of thermal expansion must be accounted for, these two thermal coupling terms become more complex with respect to \(\tau_l\) and new terms also arise in connection with the quadratic and cubic stiffnesses, see [10].

The governing equations of the thermal generalized coordinates, i.e. describing conduction, can also be derived from thermoelasticity, see [6-10]. They involve a term accounting for the latency effect and accounts for the change in geometry induced by the deformations both of which are small especially in comparison with the convection from the fluid. Since no aerodynamic model is implemented here, the temperature distribution will be assumed given, i.e. from [4], and will be projected directly on the representation of Eq. (1). Thus, the conduction equation will not be solved and accordingly no further detail in regards to it will be presented.

The above discussion has provided the parametric form of the reduced order model governing equations. Given the basis functions \(T^{(n)}\) and \(\varphi^{(n)}\), it is necessary to determine the parameters \(M_{ij}, K_{ij}^{(1)}, K_{ijl}^{(2)}, K_{ijlp}^{(3)}, K_{ijl}^{(th)},\) and \(F_{il}^{(th)}\) from the finite element model of the panel. This effort is typically accomplished first in the absence of temperature to determine the parameters \(K_{ij}^{(1)}, K_{ijl}^{(2)},\) and \(K_{ijlp}^{(3)},\) see discussion in [6] and the recent tangent-stiffness based alternative of [14]. Once the temperature independent parameters are identified, temperature distributions proportional to each of the thermal modes are imposed in turn and
the coefficients $K_{ijl}^{(th)}$ and $F_{il}^{(th)}$ are determined, see [6-10]. Finally, the mass matrix $M$ is determined as in linear structural dynamics problems.

3. PANEL PROPERTIES AND FINITE ELEMENT MODEL

The basic dimensions of the panel considered here are given in Fig. 1. The skin is 0.065 inches thick while the stiffeners attached at either side, are only half as thick, i.e. 0.0325 inches thick. The composite panel has properties of advanced carbon-carbon 4, which are shown in Table 1.

The finite element model is composed of 4 node plate elements in Nastran (CQUAD4) which are squares with sides 0.25 inches long. There are 2400 elements and 2499 nodes. Further, the boundary conditions enforced on the panel are:

(i) at the leading edge, $x=0$ and $z=0$: zero displacements along both $x$ and $z$ and zero rotations in all three directions.

(ii) at the trailing edge, $x=12$ in and $z=0$: zero displacements along $z$ and zero rotations in all three directions. Additionally, springs acting in the $x$ direction with a spring constant of 2378 lb/in are attached.

(iii) at center of the panel: zero displacement along $y$ constraint to block the rigid body motions.

The reference temperature of the panel is assumed to be 530 R.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>0.065</td>
</tr>
<tr>
<td>Young’s Modulus {$11$}</td>
<td>15 E6 psi</td>
</tr>
<tr>
<td>Young’s Modulus {$22$}</td>
<td>15 E6 psi</td>
</tr>
<tr>
<td>Shear Modulus {$12$}</td>
<td>2.5 E6 psi</td>
</tr>
<tr>
<td>Poisson’s Ratio {$12$}</td>
<td>0.3</td>
</tr>
<tr>
<td>CTE {$11$}</td>
<td>5.8 E-7 l/R</td>
</tr>
<tr>
<td>CTE {$22$}</td>
<td>5.8 E-7 l/R</td>
</tr>
</tbody>
</table>

Table 1. Material Properties of Panel.

4. BASIS CONSTRUCTION

4.1. Thermal Basis

The first issue to be addressed is the construction of the thermal reduced order basis following the broad framework outlined in section 2.1. Owing to the very small thickness, the temperature will be assumed to be uniform through it and thus varying only over the surface of the skin and stiffeners. The temperature fields which are to be represented are those obtained in the one-way coupled, constant rate ascent trajectory analysis of [4] with the 10% spring boundary condition, in which the flow and its convection only act on the skin (top) of the panel. Accordingly, there is no heating taking place on the stiffeners and the heat must thus flow on them from the fold lines, where skin and stiffeners meet, to the free ends. Further, in the one-way coupled analysis, the heat convection is fairly constant through the skin resulting in a reasonably uniform temperature distribution. These comments are well reflected on the temperature distribution of Fig. 2 which was obtained after 300 s of analysis. Note that Fig. 2 is a “developed” 2D image of the entire panel. The stiffeners on the side have been “folded” upwards so that they lie in plane with the skin of the panel. The $y$ domain from 0 to 10 inches represents the skin of the panel, while the regions less than 0 and greater than 10 represent the two stiffeners. The flow moves from left to right. The variation of temperature in the direction of the flow is mild. It is also in the $y$ direction except in the stiffener domains where it is severe.
The development of the thermal basis was carried out as broadly described in section 2.1. The eigenvectors of the generalized conductance capacitance eigenvalue problem of the entire panel were first determined. To assess the applicability of this basis, the temperature distribution of Fig. 2 was first projected on these eigenvectors. However, it was found that the most influential eigenvectors from this basis set did not produce fast enough convergence onto the desired temperature field. This result is not surprising: the sharp temperature gradients on the stiffeners are difficult for this basis set to capture.

This finding is consistent with the discussion of section 2.1, i.e. that the thermal basis must adapt to the interaction with the aerodynamics and the convection. The need for such an adaptation on the stiffeners is clear. So, in addition to the eigenvectors of the entire panel, another set of modes was generated from the conductance-capacitance generalized eigenvalue problem of the panel when the fold line connecting the skin and the panel was constrained to constant temperature.

In total, 13 eigenvectors from the unconstrained panel model and 8 eigenvectors from the constrained panel model were selected for the final thermal ROM basis. Figure 3 shows the temperature representation error

\[ \varepsilon_{rep,T} = \frac{\| T - T_{proj} \|}{\| T \|}. \]

obtained by subtracting from the temperature distribution its projection on the thermal basis, as in Eq. (1). This error was computed from the data of [4] at every 50 seconds of the 300 seconds of analysis. Although the error increases with time, it remains very low throughout, less than 0.5% and thus suggests that the thermal basis developed above is applicable for the one-way coupled analysis of the panel of Fig. 1.

Since this final temperature is the most difficult to represent, the model will be validated in subsequent discussions using this final temperature field only. Figure 4 shows the representation error of this temperature field decreasing as it is projected onto increasingly more modes of the 21 mode thermal basis.
4.2. Structural Basis

The construction of an uncoupled structural basis was addressed in [10] and led to a 32 mode structural model involving both linear and dual modes. The former were selected based on an excitation band of 2kHz for the planned structural-thermal-aerodynamic-acoustic computations.

As discussed in section 2.1, it was necessary to enrich the structural-only basis to account for the displacements induced by the thermal expansion. This construction and its validation were performed in two different steps. It was assumed in a first step that the temperature distribution was uniform and the single enrichment corresponding to the static linear response to this uniform temperature change was determined. This effort thus led to a 33 structural mode basis.

Next, structural basis enrichments were developed to capture the thermal expansion induced by the remaining 20 thermal modes of the thermal basis of section 4.1. Since adding 20 modes to the basis was not desirable, a POD analysis was performed on these 20 displacement sets and the first 3 POD modes were found to sufficiently improve the ability of the structural basis to represent the desired displacement field. The structural basis included these three modes in addition to the 33 modes previously described. The resulting 36 mode reduced order model gave a transverse error of 0.62 % and inplane error of 0.19% when the temperature field alone was applied to the model.
5. ROM PREDICTION AND COMPARISONS

5.1. Uniform Temperature Validation

The validation of the structural reduced order models with bases constructed in the previous sections was carried out first under uniform temperature fields and without applied pressure. Shown in Figs 5 (a) and (b) are the predicted transverse displacement and magnitude of inplane displacement, respectively, by the reduced order model while the corresponding figures for the Nastran predictions are on Figs 6. The matching between these two sets of figures is excellent, the transverse norm error is 0.54% and the inplane one is 0.05%. It is interesting to note that the inplane response at the center of the panel near the leading edge is close to zero, which is intuitive given the boundary conditions. The magnitude of the inplane displacement grows larger near the edges, where thermal expansion has caused the most displacement. The discontinuity at the stiffener is due to the expansion in the $y$ direction being defined as transverse for the stiffener vs. inplane for the skin. This is also the reason for the sudden appearance of large transverse motion in the stiffener in Fig. 5 (b). Positive transverse motion in the stiffener is defined as motion away from the skin, as opposed to motion that would place the stiffener under the skin.

![Figure 5](image1.png)

Figure 5. (a) Magnitude of inplane displacement and (b) transverse displacement from uniform temperature field of 2700 R. Units in inches. Results from 33 mode ROM.

![Figure 6](image2.png)

Figure 6. (a) Magnitude of inplane displacement and (b) transverse displacement from uniform temperature field of 2700 R. Units in inches. Results from Nastran nonlinear.
Additional comparisons between responses predicted by the ROM and Nastran were carried out with the uniform temperature increase of 2700 R when the panel skin (not stiffener) was also subjected to a uniform pressure, see Table 2 for error comparisons and Figs. 7 and 8 for an upward pressure load of 3 psi. Again, an excellent matching between Nastran and ROM results is obtained with displacements varying between approximately -3 and +3 thicknesses.

<table>
<thead>
<tr>
<th>Transverse error [%]</th>
<th>Inplane error [%]</th>
<th>Nastran center disp. [thick.]</th>
<th>ROM center disp. [thick]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 psi down</td>
<td>.38</td>
<td>-2.98</td>
<td>-2.98</td>
</tr>
<tr>
<td>2 psi down</td>
<td>.29</td>
<td>-2.49</td>
<td>-2.49</td>
</tr>
<tr>
<td>1 psi down</td>
<td>.49</td>
<td>-1.80</td>
<td>-1.81</td>
</tr>
<tr>
<td>0 psi</td>
<td>.54</td>
<td>-0.065</td>
<td>-0.064</td>
</tr>
<tr>
<td>1 psi up</td>
<td>.51</td>
<td>1.62</td>
<td>1.62</td>
</tr>
<tr>
<td>2 psi up</td>
<td>.33</td>
<td>2.25</td>
<td>2.25</td>
</tr>
<tr>
<td>3 psi up</td>
<td>1.6</td>
<td>2.69</td>
<td>2.70</td>
</tr>
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</table>

Table 2. Results from 2700 R uniform temperature and pressure loads. 33 mode ROM
5.2. Non-uniform Temperature Validation

The validation of the enriched structural basis proceeded finally with the 36 mode model of section 4.2 and the temperature field of Fig. 2 projected on the 21 thermal mode basis. The displacement field induced by this temperature distribution without and with additional uniform pressure on the panel skin was computed by the ROM and by Nastran, see Figs 9-12 and Table 3. The observations drawn in connection with the uniform temperature are found to be applicable again here: an excellent matching between ROM and Nastran predictions is consistently observed.

![Figure 9](image1.png)  
(a) Magnitude of inplane displacement and (b) transverse displacement from final temperature field. Units in inches. Results are from 36 mode ROM.

![Figure 10](image2.png)  
(a) Magnitude of inplane displacement and (b) transverse displacement from final temperature field. Units in inches. Results from Nastran nonlinear.

<table>
<thead>
<tr>
<th>Pressure</th>
<th>Transverse error [%]</th>
<th>Inplane error [%]</th>
<th>Nastran center disp. [th]</th>
<th>ROM center disp. [th]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 psi down</td>
<td>.35</td>
<td>.37</td>
<td>-2.99</td>
<td>-2.99</td>
</tr>
<tr>
<td>2 psi down</td>
<td>.32</td>
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<td>-2.50</td>
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</tr>
<tr>
<td>1 psi down</td>
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<td>-1.81</td>
<td>-1.82</td>
</tr>
<tr>
<td>0 psi</td>
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<td>.19</td>
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<td>.028</td>
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<td>3.77</td>
<td>2.72</td>
<td>2.74</td>
</tr>
</tbody>
</table>

Table 3. Results from final temperature with pressure loads. 36 mode ROM
It was finally desired to confirm the adequacy of the 36 modes over the entire temperature range. To this end, it was used for the prediction of the panel response in the absence of thermal loading but with applied pressure. Shown in Figs 13 and 14 (a) and (b) are the transverse and inplane responses to a loading of 3 psi in the upward direction, while Figs 15 and 16 (a) and (b) show the transverse and inplane response to a downward loading of 3 psi. Moreover, the errors between the ROM and Nastran predictions for various pressure loads acting on the panel without a thermal load are shown in Table 4. They are consistent with prior errors, see Tables 2 and 3.

<table>
<thead>
<tr>
<th></th>
<th>Transverse error [%]</th>
<th>Inplane error [%]</th>
<th>Nastran center disp. [th]</th>
<th>ROM center disp. [th]</th>
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<td>3 psi down</td>
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<tr>
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<td>.58</td>
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</tr>
<tr>
<td>1 psi down</td>
<td>.18</td>
<td>.70</td>
<td>-1.20</td>
<td>-1.20</td>
</tr>
<tr>
<td>1 psi up</td>
<td>.16</td>
<td>1.24</td>
<td>1.14</td>
<td>1.14</td>
</tr>
<tr>
<td>2 psi up</td>
<td>.62</td>
<td>1.36</td>
<td>1.81</td>
<td>1.82</td>
</tr>
<tr>
<td>3 psi up</td>
<td>2.33</td>
<td>3.91</td>
<td>2.28</td>
<td>2.31</td>
</tr>
</tbody>
</table>

Table 4. Results from uniform pressure loads without thermal loading. 36 mode ROM
6. CONCLUSION

The construction of the thermal and structural bases in coupled structural-thermal reduced order models of a representative hypersonic vehicle panel was detailed and validated. The thermal basis was constructed using the eigenvectors of three separate heat conduction problems that relate to the entire panel and its two subcomponents: skin and stiffeners. The structural basis was constructed by enriching the one developed previously for purely mechanical loading with linear static responses of the structure to temperature distributions proportional to each of the thermal modes. The resulting structural-thermal reduced order model was shown to accurately predict the structural displacements induced by a combination of thermal and pressure loads when compared to the full Nastran results. The range of pressure loading applied to the structure was from 3 psi down to 3 psi up, with the maximum temperature applied to the structure being approximately 2700 R. This resulted in a maximum transverse displacement of about 3 thicknesses. The ROM provided accurate predictions in all cases tested, with the largest error being under 4% for the highest upward pressure loading case.
Figure 15. (a) Magnitude of inplane displacement and (b) transverse displacement from 3 psi downward loading. Units in inches. Results are from 36 mode ROM.

Figure 16. (a) Magnitude of inplane displacement and (b) transverse displacement from 3 psi downward loading. Units in inches. Results from Nastran nonlinear.

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