DYANMICS OF A NONLINEAR LIQUID SLOSHING INSIDE A TANK

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1. INTRODUCTION

Sloshing is a phenomenon which is caused by the movement of the container. If the excitation frequency is close to the natural frequency of the container, the sloshing can be violent, the structure of the container would damage under the heavy shock load on the wall, and the hydrodynamic forces and moments acting on the wall would change the dynamic behavior of the container and its host systems in return. Hence, sloshing will have much impact on the safety and stability of the system. It is important to study the dynamic characteristics of the sloshing in moving containers.

In the past a few years, research of sloshing has been made by using analytic, numerical and experimental methods (e.g., Abramson [1], Faltinsen [2], Ibrahim [5]). Because of the rapid development of electronic computers, it seems that numerical simulation has become a mainstream method (e.g., Wu, etc[4], Pirker.S, etc[6]). FLUENT, which is an excellent CFD(Computational Fluid Dynamics) software, can be used to simulate many complicated fluid dynamical processes with high precision, so it has been widely applied in CFD problems.

In this paper, liquid sloshing problems with free surface in moving containers are studied, the CFD models of liquid sloshing are built and calculated, taking consideration of many factors such as tank shapes, mesh dividing and convergence, which affect the calculating results. The movements of containers are simulated by using dynamic mesh technique, and UDF (User Defined Function) is programmed to
get the hydrodynamic forces and moments. Natural frequencies of sloshing in tanks with different shapes or different filling-rates are studied, and the nonlinear impact of excitation on frequencies and amplitudes of sloshing forces are analyzed. The results will provide theoretical basis for the optimal design and proper operation of tanks, and lay the foundation for the more research of dynamic coupling of sloshing in tanks and its host systems.

2. CFD MODELS OF 3D NONLINEAR LIQUID SLOSHING

In order to build CFD models of liquid sloshing in FLUENT, there are three problems to be investigated: dynamic mesh technique, mesh dividing, and solver settings.

2.1 Dynamic mesh technique

The phenomenon of sloshing is induced by the movement of containers. So, it is convenient to calculate sloshing problems in moving coordinate systems. In a non-inertial coordinate system, the form of Navier-Stokes equation must be changed:

\[
\frac{du}{dt} + 2\omega \times u + \frac{d\omega}{dt} \times r + \omega \times (\omega \times r) + \frac{dV}{dt} = F - \frac{1}{\rho} \nabla p + \frac{\mu}{\rho} \Delta u
\] (1)

where \( u \) is the velocity vector of the fluid, \( \omega \) is the angular velocity of the moving coordinate systems, \( V \) is the velocity vector of the moving coordinate systems, \( F \) is the mass force, and \( r \) is the radius vector in moving coordinate systems.

Navier-Stokes equation can be solved only in inertial coordinate systems in FLUENT. In order to solve this problem, some articles (e.g., Shang, etc[9]) used the method of exerting the reverse inertia force to the sloshing fluid. In this paper, dynamic mesh technique is adopted. The dynamic mesh model in FLUENT can be used to model flows with moving boundaries. So the sloshing problems can be calculated in the inertial coordinate system, and no reverse inertia force needs to be exerted. The containers are subjected to horizontal harmonic vibration, so UDF is programmed to simulate the movement of the wall in containers. In order to update the volume mesh in the deforming regions, spring-based smoothing method and local remeshing method are used.

2.2 Mesh dividing

The liquid storage tanks in tank trucks are studied. 3D CFD models with different shapes are built. The shapes of cross-section are circle, ellipse, and Reuleaux triangle, which is designed by Kang, etc[7]. The radius of the circle is 364mm; the long and the minor axis of the ellipse are 445mm and 297mm, respectively; the width and the height of the Reuleaux triangle are 774.45mm and 691.98mm, respectively. So the three cross-sections have the same area. The longitude of the three tanks is set to be 1850mm.

The meshes are divided in GAMBIT. First, the cross-sections of the tanks are divided by Quad-Pave scheme, so the unstructured surface mesh grids including quadrilateral unit are generated. Second, the grids are projected along the longitudinal direction by Hex/Wedge Cooper scheme, so the space mesh grids are derived.
In order to investigate the relationship between the number of grids and accuracy of the results, the grid size is set to be 10mm and 20mm. The sloshing forces are calculated. The two forces are nearly the same. But the number of grids of size 10mm is fifteen times more than the number of grids of size 20mm; and much more calculation time is needed. So the grid size 20mm is selected, and the number of grids is 105648.
2.3 Solver settings

1. The standard $k-\varepsilon$ two-equation model of turbulence is selected, where $k$ is the turbulent kinetic energy and $\varepsilon$ is turbulent dissipation rate. In the areas affected by the viscosity, wall function is used to connect the quantities in the wall with the quantities in the turbulence. So the mesh refinement is not necessary in the wall. Because of its stability, the standard wall function is selected.

2. In the CFD models, Boolean operation is carried out to delete the wall of the tanks. The solid-liquid interface is set to be the wall boundary condition, the free surface of liquid is set to be the pressure outlet condition.

3. Pressure-based coupled solver of incompressible flow is selected. NITA (Non-Iterative Time Advancement) scheme is used to improve speed and efficiency of calculation, and Green-Gauss based discrete scheme is used to improve precision of tank grids.

4. VOF (Volume of Fluid) model based on Eulerian coordinates is used to trace the liquid free surface, and implicit body force equation is used to improve convergence.

5. The residual error of velocity components, pressure components, and VOF is set to be 0.001.

   In the UDF, the macro DEFINE_CG_Motion is used to define the velocity and angular velocity of the tanks, the macro Compute_Force_And_Moment is used to calculate the hydrodynamic forces and moments on the wall.

3. INVESTIGATION OF NATURAL FREQUENCIES IN TANKS

   The first natural frequency of the half-filled tank with circular cross-section is derived in this way:

   1. In FLUENT, wall boundaries are imposed an excitation of velocity as:

      $$v(t) = \begin{cases} 
      0, & t > 0.01 \\
      0.5, & 0.01 < t \leq 0.01 \\
      0, & t \leq 0.01 
      \end{cases}$$  \hspace{1cm} (2)

      After 0.01s, the excitation is cancelled; the kinetic process is free sloshing of liquid.

   2. The macro Compute_Force_And_Moment calculates the sloshing force on the wall, and then spectral analysis is carried out in MATLAB. In order to compare the results with other papers (Papaspyrou S, etc[8]), the radius is set to be variable. First, the radius is 0.1m; the sloshing force and response spectrum are showed in Fig.2.

![Fig.3. Sloshing force and response spectrum of the half-filled tank](image-url)
Papaspyrou S, etc[8] derive an analytic approximate solution to the first natural frequency of the half-filled tank with circular cross-section. In this paper, numerical solutions are calculated in FLUENT with different radii, and compared with the analytic approximate solutions. The results are showed in Tab.1. The errors turn out to be small; the numerical simulation is accurate and efficient.

<table>
<thead>
<tr>
<th>Radius(m)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytic solution(Hz)</td>
<td>1.71</td>
<td>1.21</td>
<td>0.99</td>
<td>0.86</td>
<td>0.77</td>
<td>0.7</td>
<td>0.65</td>
<td>0.6</td>
<td>0.57</td>
<td>0.54</td>
</tr>
<tr>
<td>Numerical solution(Hz)</td>
<td>1.85</td>
<td>1.31</td>
<td>1.07</td>
<td>0.93</td>
<td>0.83</td>
<td>0.73</td>
<td>0.68</td>
<td>0.63</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>Relative error</td>
<td>8.2%</td>
<td>8.3%</td>
<td>8.1%</td>
<td>8.1%</td>
<td>7.8%</td>
<td>4.3%</td>
<td>4.6%</td>
<td>5%</td>
<td>1.7%</td>
<td>7.4%</td>
</tr>
</tbody>
</table>

Tab.1. Comparison between the numerical solutions and the analytic approximate solutions

The natural frequencies of various tanks in Fig.1 are calculated, with different filling-rates. The results are shown in Tab.2. For the tanks with the same shape of cross-section, the higher the filling-rate, the larger the natural frequency. For the same filling-rates, natural frequencies of tanks with elliptical cross-section are the smallest.

<table>
<thead>
<tr>
<th>Shapes of cross-section</th>
<th>30</th>
<th>50</th>
<th>70</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>circle</td>
<td>0.88</td>
<td>0.93</td>
<td>1.02</td>
<td>1.26</td>
</tr>
<tr>
<td>ellipse</td>
<td>0.72</td>
<td>0.81</td>
<td>0.91</td>
<td>1.14</td>
</tr>
<tr>
<td>Reuleaux Triangle</td>
<td>0.78</td>
<td>0.97</td>
<td>1.10</td>
<td>1.29</td>
</tr>
</tbody>
</table>

Tab.2. Natural frequencies of tanks with different shapes and different filling-rates

4. INVESTIGATION OF SLOSHING FORCES IN TANKS

The movement of tanks will cause the sloshing of liquid, so the sloshing forces are worthy of attention. The liquid in tanks is imposed an excitation of acceleration along the traverse direction:

\[ a = \sin(2\pi ft) \quad (m/s^2) \quad (3) \]

The traverse hydrodynamic forces are calculated, and Fourier Transform is employed to get the forces’ response spectrum. If the excitation is along the traverse direction, the longitudinal and vertical hydrodynamic forces are small, so only traverse sloshing forces are considered.

4.1 The influence of excitation on frequencies of sloshing forces

First, non-resonance situation is investigated. Take the half-filled tank of Reuleaux Triangle as an example. The natural frequency is 0.97Hz. The excitation frequencies
are 0.3Hz and 3Hz. Fig.3 shows that when the natural frequency is close to the excitation frequency, the natural frequency exerts an influence on the sloshing forces; otherwise, the effect of the natural frequency is small.

![Figure 3](image_url)

Fig.4. Response spectrum of sloshing forces in various excitation frequencies

Then, resonance situation is investigated. The shape of the cross-section is still Reuleaux Triangle, and filling-rates are 30% and 70%. The natural frequencies are 0.78Hz and 1.10Hz. Fig.4 shows that the response spectrum is made up of fundamental frequency and high harmonics which are odd times of fundamental frequency. In the case of resonance, sloshing forces display strong nonlinear features.

![Figure 4](image_url)

Fig.5. Response spectrum of sloshing forces in resonance

4.2 The influence of excitation on amplitudes of sloshing forces

Various excitation frequencies are selected, sloshing forces of time domain are calculated, and took the average when the sloshing forces are in steady states. The shape of the cross-section is Reuleaux Triangle, and filling-rates is 50%. The range of excitation frequencies is from 0.1~4Hz. Fig.5 displays some examples of sloshing forces in various excitation frequencies.

Fig.6 shows how the amplitudes of sloshing forces change with the excitation frequencies. The graph’s horizontal axis shows the frequency ratio, ie, the ratio of excitation frequency to the natural frequency. The vertical axis shows the sloshing forces. When the excitation frequency is small, the amplitude of the sloshing forces is around a constant. When the excitation frequency ratio is close to 1, the sloshing forces increase sharply before the peak, and reduce sharply after the peak. Because of
the effect of liquid damping, the frequency ratio of resonance is slightly less than 1. If the frequency ratio is $\sqrt{2}$, the sloshing force is minimized.
5. CONCLUSIONS

In this paper, sloshing problems in tanks are investigated. The CFD models of liquid sloshing are built, and numerically simulated in FLUENT, and hydrodynamic forces on the wall are calculated. The natural frequencies of sloshing in tanks with different shapes or different filling-rates are derived, and the impact of excitation on frequencies and amplitudes of sloshing forces are analyzed. The following conclusions can be drawn:

(1) Natural frequency of sloshing is an important characteristic of liquid dynamics. The shape and the filling-rate of tanks have great influence on the natural frequencies.
(2) If the excitation frequency is close to the natural frequency, the sloshing forces increase sharply, and strong nonlinear phenomenon occurs.
(3) When the ratio of excitation frequency to the natural frequency is larger than $\sqrt{2}$, the amplitudes of sloshing forces are small, so they can be controlled within a certain range.

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