IDENTIFICATION OF VIBRATION PATH IN A GASOLINE DIRECT-INJECTION ENGINE USING TWO INPUT-ONE OUTPUT MODEL

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ABSTRACT
The paper describes the method of separating the effect of two vibration inputs on the total vibration output from an engine block of a car. The authors use the partial and pure coherence functions, which are derived from the power spectral functions of the input and output vibrations, to distinguish the effects of the high-pressure pump from those of the engine itself. The experimental measurements and spectral analysis show that the effects of the pump dominate in a wide frequency range, though the engine and the pump have nearly the same effects at around 1000 Hz.

1. INTRODUCTION
Due to advances in the automobile industry in recent years, today’s vehicles are becoming increasingly luxurious and offer enhanced performance. Accordingly, consumers are shifting their interest from output and fuel efficiency to vibration, noise, and ride quality. While numerous studies have been conducted on gasoline engines [1, 2], gasoline direct-injection (GDI) engines require further research. Vibration and noise from a pump due to high pressure occurring in a GDI engine can be felt inside the vehicle, leading to complaints from customers. Therefore, an analysis technology to determine the contributions of vibrations coming from GDI engine parts must be developed. A GDI engine includes a high pressure pump, a fuel rail, an injector, and a pressure sensor. The noise and vibration caused by these parts are perceived inside the vehicle as a mixture of the parts’ respective vibration, necessitating analysis of the parts’ respective vibration paths and levels of contribution. Noise and vibration from GDI engine parts do not seem to be generated independently. Rather, they mostly exist in a combined form, which makes it difficult to analyze each part’s vibration level when examining the effect of a specific source of vibration. In the case of an independent system, spectral frequency analysis is possible by applying the existing frequency response function method. However, the noise and vibration analysis of GDI engine parts has limitations due to correlations among the parts. For such a case, it is easier to use a partial coherence function that considers correlations between each source of input and output, and analyzes their respective effects [3]. We propose a new model to quantify the contributions of GDI engine parts to the high pressure pump vibration that is a major vibration source among GDI engine parts. A model was developed based on partial coherence theory. A validation test for the proposed model was performed, and the model was applied to the analysis of high pressure pump vibration.
2. SPECTRAL ANALYSIS FOR TWO-INPUT/SINGLE-OUTPUT SYSTEM

An ordinary coherence function is used when there is no correlation between input sources and when the sources are independent from one another. However, the main parts of the GDI engine for this study are a high pressure pump, an injector, a fuel rail, and a pressure sensor, which have correlations among them [5]. Therefore, these engine parts should be modeled to have multiple inputs and a single output, as shown in Fig. 1, rather than a single input.

![Figure 1. Multi-input/single-output model of a dynamic system for vibration path analysis.](image)

In addition, they need to be analyzed using the multi-dimensional spectral method. In this study, the parts were modeled with a two-input/single-output system [6], as shown in Fig. 2, based on the multi-input/single-output system.

![Figure 2. Two-input/single-output model of a dynamic system with two independent inputs.](image)

Through a partial coherence function (one of the multi-dimensional spectral analytical methods), the contribution of each multiple input was analyzed [7]. If outputs are shown as results of mutual interference between the vibration paths of the GDI engine parts, as shown in Fig. 3, their respective actual levels of contribution can be identified by removing correlations between parts.

![Figure 3. Two-input/single-output model of a dynamic system with partially correlated inputs.](image)

2.1 Ordinary Coherence Function

The portion taken up by each input for the entire output can be expressed as an ordinary coherence function \( \gamma_{xy}^2(f) \), as shown in Eq. (1), by using the auto-spectrum and cross-spectrum [6-7] as follows:

\[
\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_x(f)G_y(f)} \quad i=1,2,\cdots,q
\]
In Fig. 2, inputs \( X_1(f) \) and \( X_2(f) \) against output \( Y(f) \) are modeled, where \( H_1(f) \) and \( H_2(f) \) are frequency response functions. Here, \( X_1(f) \) and \( X_2(f) \) are the values resulting from the Fourier transformation of \( x_1(t) \) and \( x_2(t) \), respectively. The ordinary correlation between the input value \( X(f) \) and the output value \( Y(f) \) can be expressed by Eq. (1). In this study, \( i \) is 1 and 2 since two inputs are used. However, if outputs are shown as results of mutual interference between the vibration paths of the GDI engine parts, as shown in Fig. 3, it is difficult to know the actual level of contribution by the input of interest. In the system as shown in Fig. 3, the correlation of the input \( X_1(f) \) does not consider the mutual link of the other input \( X_2(f) \). Hence, a partial coherence function from which correlation properties are eliminated was used in a multi-input system.

### 2.2 Ordinary Coherence Function

The output power spectrum \( G_{yy}(f) \) of the multi-input/single-output system as shown in Fig. 1, which is a multi-input system, can be calculated as

\[
G_{yy}(f) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \text{H}^*_i(f) \text{H}_j(f) G_{ij}(f) + G_{nn}(f)
\]  

(2)

where \( \text{H}_i(f) \) is a frequency response function that contributes to the \( i \)-th system, \( \text{H}^*_i(f) \) is a complex conjugate number of the \( \text{H}_i(f) \) function, and \( G_{nn}(f) \) is the noise power spectrum. In this case, the contribution by any input against its output can be calculated using the frequency response function method. In the case of the system as shown in Fig. 3, it is difficult to determine the extent to which each input value contributes to output values. Therefore, the output spectrum can be expressed as a residual spectrum from which the linear effects of inputs on outputs are eliminated:

\[
G_{ij'} = G_{ij'(r-1)}(f) - L_{ij}(f) G_{ij'(r-1)}(f)
\]

(3a)

\[
L_{ij}(f) = \frac{G_{ij'(r-1)}(f)}{G_{rr'(r-1)}(f)}
\]

(3b)

where, \( L_{ij}(f) \) is the optimal transfer function between the \( r \)-th input and \( j \)-th outputs when the correlation properties between the \( r \)-th input and \( j \)-th outputs is eliminated. The residual spectrum between the input \( X_2(f) \) and the output \( Y(f) \), from which the effect of \( X_1(f) \) is eliminated from the two input/single-output model, is expressed as follows [6]:

\[
G_{2y1}(f) = E\left[ X^*(f) Y(f) \right] = G_{2y}(f) - \left( \frac{G_{11}(f)}{G_{11}(f)} \right) G^*_{12}(f)
\]

(4)

The auto spectrum for input \( X_2(f) \), from which the effect of \( X_1(f) \) is eliminated, is given by

\[
G_{221}(f) = E\left[ X^*_{21}(f) X_{21}(f) \right] = G_{22}(f) - \frac{G_{12}(f) G^*_{12}(f)}{G_{11}(f)} = G_{22}(f) (1 - \gamma^2_{12}(f))
\]

(5)

The auto spectrum for output \( Y(f) \) from which the effect of \( X_1(f) \) is eliminated is given by
\[ G_{yy1}(f) = G_{y2}(f)(1 - \gamma^2_{1y1}(f)) \] (6)

The partial coherence function of \( X_2(f) \), from which the relation \( X_1(f) \) is eliminated, can be expressed as [7]

\[ \gamma^2_{2y1}(f) = \frac{|G_{2y1}(f)|^2}{G_{221}(f)G_{yy1}(f)} \] (7)

Figure 3 shows a model of the contribution of \( X_1(f) \) and \( X_2(f) \) to output \( y \). Unlike Fig. 2, the model in Fig. 3 was made by considering correlations between each input value. In other words, the input of the excitation forces of \( X_1(f) \) and \( X_2(f) \) indicates the output vibration of \( Y(f) \), whereas the input of \( X_1(f) \) goes into \( y \) through \( X_2(f) \). In this case, the actual level of contribution by \( X_1(f) \) should be calculated with a spectrum without the relation \( X_2(f) \) and by using a partial coherence function. To actually detect a vibration source, a coherence output spectrum that calculates the pure level of contribution of the input source to the output must be obtained. A coherence output spectrum is used to identify the amount of power at the output generated by a pure input value that is free of linear effects related to the input value. To obtain a partial coherence output spectrum of the two-input/single-output model, Eq. (8) can be used as follows:

\[ G_{y12}(f) = \gamma^2_{1y2}(f)G_{y2}(f) \] (8a)

\[ G_{y21}(f) = \gamma^2_{2y1}(f)G_{yy1}(f) \] (8b)

Here, \( G_{y12}(f) \) is the pure coherence output spectrum of \( X_1(f) \) from which the input \( X_1(f) \) contributes to the output \( Y(f) \), and the correlation of the input \( X_2(f) \) is eliminated in this case. Also, \( \gamma^2_{1y2}(f) \) is the partial coherence function of the input \( X_1(f) \), or pure correlation free of the correlation \( X_2(f) \), and \( G_{y2}(f) \) is the output power from which the correlation of \( X_2(f) \) for the \( Y(f) \) output power is eliminated. The coherence output spectrum of \( X_1(f) \) can be obtained by multiplication of the partial coherence function of \( X_1(f) \) with \( G_{y2}(f) \), which is the output power.

3. EXPERIMENTAL VALIDATION OF TWO-INPUT/SINGLE-OUTPUT SYSTEM MODEL

3.1 Validation Test

In order to validate the proposed model in our application to engine vibration path identification, a rig test was performed using a prototype pump model and the cylinder head of a practical engine. Two shakers were used to excite the test system. One is a larger power shaker exciting the cylinder head, and the other is a small power shaker exciting the pump model as shown in Fig. 4 shows the experimental equipment used for this test. The signals of the two input forces were generated by an output channel of LMS Mobile system (LMS Mobile 32 channel, LMS Corporation Belgium) and were enhanced by an amplifier (Dynamics 200, Angel Corporation, Korea). Two sinusoidal signals were generated for the excitation signals by the LMS Mobile system. In order to excite the middle of the pump model and the cylinder head independently without correlation between two signals, the small shaker was used to excite the middle of the pump model in the x-direction with a sinusoidal signal at a frequency of 2k Hz. The large shaker excited the cylinder head in the z-direction with a sinusoidal signal at a frequency of 700 Hz. The acceleration generated by the excitation
forces of the two shakers propagated to the top of the pump model through the engine and the middle of the pump model, as shown in the Fig. 4.

![Figure 4](image)

Figure 4. Experiment set-up for identification of partial coherence.

$X_1(f)$ is the Fourier-transformed version of the vibration signal measured at the engine excitation position. $X_2(f)$ is the Fourier-transformed version of the vibration signal measured at the middle point of the pump model. $Y(f)$ is the Fourier-transformed version of the vibration signal measured at the top point of the pump model. Engine vibration input $X_1(f)$ and pump vibration input $X_2(f)$ are independent since their excitation directions are different, and the frequency of the excitation signal is also different. The vibration signal of input $X_1(f)$ was measured by a sensor attached at the engine excitation position. The vibration signal of input $X_2(f)$ was measured by a sensor attached to the excitation position of the pump prototype model. The sensors used for this test are tri-axial accelerometers (Type 64-10, ENDEVCO, USA). Output vibration signal $Y(f)$ is measured by the sensor located on the top of the pump prototype model. The measured acceleration signals were recorded, transferred to the LMS Mobile system, and amplified by the built-in amplifier of the LMS Mobile system. These acceleration data were analyzed by using LMS Test-Lab.

![Figure 5](image)

Figure 5. Multi-input/single-output modeling for vibration path analysis of a pressure pump.

MATLAB® software (version 7.0, MathWorks, USA) was used to calculate ordinary coherence and partial coherence for the signals. The vibrations at the three points were also simultaneously measured by using three accelerometers. The sampling frequency of the acceleration date was 12.8 KHz. To apply the proposed model to the validation test, the
vibration transfer system, as shown in Fig. 5(a), was simulated as a two input/one output system model as shown in Fig. 5(b). Engine vibration $X_1(f)$ was transferred to the top position of the pump model through the middle position of the pump. The vibration $X_2(f)$ at the middle position of the pump model was also transferred to the top position of the pump model. Therefore, output vibration $Y(f)$ at the top of the pump model was affected by two vibration inputs. The vibration in the middle of the pump was affected by engine vibration. In order to quantify the contribution of each input, we calculated the partial coherence between output $Y(f)$ and each input $X_1(f)$ or $X_2(f)$. Fig. 6 shows the ordinary coherences between output $Y(f)$ and each input $X_1(f)$ or $X_2(f)$. At frequencies of 700, 1400, and 2000 Hz, ordinary coherence is high, which is a natural result.

![Figure 6. Ordinary coherence between engine ($X_1(f)$) and pressure pump-up ($Y(f)$), and ordinary coherence between pump-mid ($X_2(f)$) and pressure pump-up ($Y(f)$).](image)

In order to exclude the interaction between engine vibration and the vibration at the middle of the pump from the output, the partial coherence between the output and input $X_1(f)$ or $X_2(f)$ was calculated as shown in Fig. 7.

![Figure 7. Partial coherence between engine ($X_1(f)$) and pressure pump-up ($Y(f)$), and Partial coherence between pump-mid ($X_2(f)$) and pressure pump-up ($Y(f)$).](image)

According to the results shown in the figure, the partial coherence between output $Y(f)$ and input $X_1(f)$ is high at 700 Hz and low at 2000 Hz. In the same fashion, the partial coherence between output $Y(f)$ and input $X_2(f)$ is high at 2000 Hz and low at 700 Hz. For the validation test, the partial coherence can be used to quantify each contribution of two coherent inputs with respect to the one output.
4. APPLICATION

4.1 Experimental Method and Set-up

We performed an experiment to quantifying the respective contributions of the engine source and pump-mid source to the vibration at the top of pump. Pump-mid means the middle position of pump. A test vehicle was installed on a chassis dynamo, and a Gamma GDI 2L engine was used. The experiment started with the engine at a steady and idle state. Then, the rotational speed was increased from 1000 rpm to 6000 rpm with step increment of 100rpm. At each increase in speed, data was collected for 10 s. Our results are based on data measured at 5,000 rpm. The input signals were measured by attaching one-directional axial accelerometers (Type 65-10, ENDEVCO, USA) to five measurement points (the engine, high pressure pump, injector, fuel rail, and pressure sensor), as shown in Fig. 8.

![Figure 8. Position of accelerometers installed on 2L Gamma GDI engine for the measurement of the vibration path.](image)

4.2 Data Processing and Analysis

As shown in Fig. 9, the data were measured by using the LMS Mobile system with 32 channels.

![Figure 9. Experiment set-up for identification of vibration path in Gamma gasoline direct-injection (GDI) engine.](image)

Each analog signal was converted to a digital signal, and the data were processed using MATLAB software. For the data processing sequence, various spectrum density functions were obtained, and then the ordinary coherence function, residual spectrum, partial coherence function, and the partial coherence output spectrum were calculated as shown in Fig. 10.
4.3 Analysis of Test Results

The proposed two-input/single model was used to analyze the respective contributions of the engine source and pump-mid source to the vibration at the pump-up. Pump-up means the top position of pump. Because the individual input values have correlations between them. In other words, inputs from the engine excitation force and the high pressure pump’s own excitation forces generate the output vibration at the pump, whereas the independent forces caused by the operation of the injector, fuel rail, and high pressure sensor excite the engine. The system was modeled as a multiple-input/single output as shown in Fig. 11.

However, since these forces are too small to excite the pump through the engine, we assumed that the vibration excited by these forces does not transfer to the pump through the engine. Therefore, the relationship between the engine and pump was modeled by using a two-input/single-output system. First, the pump-up vibration was divided by two contributions due to the engine excitation and pump-mid excitation. Then, the engine vibration was divided by multiple contributions due to the operating forces of the injector, fuel rail, and high pressure sensor. The actual location of the pressure pump, shown in Fig. 8, can be confirmed with the photos of the experiment shown in Fig. 12.
Figure 12. Position of sensors for measurement of pressure pump vibration.

$X_1(f)$ is the engine input source for output power $Y(f)$, which is the vibration at the pump-up, and $X_2(f)$, which is the vibration at the pump-mid. Based on this modeling, the difference between the ordinary coherence function and the partial coherence function is identified as follows. Fig. 13 shows an ordinary coherence function between input source $X_1(f)$ and output vibration $Y(f)$ as a dotted line. The ordinary coherence function between input source $X_2(f)$ and output vibration $Y(f)$ is shown as a solid line. Because mutual inputs between them were not considered in this case, it is impossible to know the actual level of contribution for each input. Moreover, it is difficult to distinguish the engine’s vibration from the vibration that comes from the high pressure pump with an ordinary coherence function alone.

Figure 13. Ordinary coherence between engine ($X_1(f)$) and pressure pump-up ($Y(f)$) and ordinary coherence between pump-mid ($X_2(f)$) and pressure pump-up ($Y(f)$)

Meanwhile, the result of the application of the partial coherence function, as shown in Fig. 14, suggests that there is a noticeable difference between the ordinary coherence function and the partial coherence function. The graph of the partial coherence function confirms that the contribution by the pump varies in the high frequency band. This is because it reflects a pure contribution by the engine to vibration that is free of the high pressure pump’s own excitation source. In other words, the partial coherence function shown in Fig. 14 expresses the pure level of contribution by each input by removing linear correlations among them.
Thus, the relationship of contributions between inputs and those between inputs and outputs of the ordinary coherence function and the partial coherence function have a high value of over 0.7. Based on this result, we assume that there is a correlation between inputs as well as between inputs and outputs. Using the residual spectra discussed in Section 2, the output spectra corresponding partial coherence functions were obtained, and coherence output spectra were made to represent the output power of a specific input for each partial coherence function. Fig. 15(a) shows the result of the partial coherence output spectrum for the pump. And Fig. 15(b) shows the result as a bar graph. In the Fig. 15(b) Etc is the injector, fuel rail, and high pressure sensor. The vibration power obtained through the coherence output spectrum, as shown in Fig. 15, is dominant on the engine around 1,000 Hz, whereas the pump-mid makes the biggest vibration contribution near 2,000 Hz.

5. CONCLUSION

In this study, a two-input/single-output system was created, and is comprised of GDI engine parts for the purpose of examining the vibration contribution of the input properties by using partial coherence functions. Our results lead to the following conclusions:

(1) When there is a correlation, as in the case of GDI engine parts, a partial coherence function that eliminates such a correlation between inputs should be used.
(2) The vibration paths and contribution of the GDI engine parts were analyzed by calculating the partial coherence functions. The vibration power obtained through the coherence output spectrum is dominant on the engine around 1,000 Hz, whereas the pump-mid makes the biggest vibration contribution near 2,000 Hz.

(3) The vibration contributions of the engine and pump were quantified for each frequency band through the calculation of the vibration power spectrum of each part based on the multi-dimensional spectral analysis method.

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