TUNEABLE VIBRATION ABSORBER FOR MULTI MODAL CONTROL OF LIGHTLY DAMPED STRUCTURES

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ABSTRACT

This paper presents the design of an experimental multi modal vibration absorber for controlling the transverse vibration of a lightly damped panel. Traditionally, when multiple modes of a structure need to be controlled many absorbers are scattered on the structure, each tuned to control a specific structural mode. However the use of multiple units increases the added weight to the structure and requires complex tuning process, which therefore raise the cost of the control treatment.

In this paper an integrated vibration absorber able to incorporate in one unit the effect of two traditional single axis absorbers is proposed. The absorber consists of a beam with a cylindrical inertial mass mounted at the centre. The connection between the mass and the beam is realised through flexible fins that allow rotational vibrations of the mass around the beam but do not allow relative displacement. When the two ends of the beam are attached to a structure, the flexural vibration of the beam-mass system produces a transverse control force on the structure while the rotational vibration of the inertial mass generates a control moment.

These two actions can be used to control the response of two resonant modes of a structure by tuning the two resonances of the device to match those of the modes under control. The flexural resonance of the beam-mass system is tuned by varying the axial tension of the beam using a piezoelectric stack transducer while the rotational resonance of the device is regulated by varying the tension on capacitors whose plates are mounting on the beam and on the rotating mass respectively.

1. INTRODUCTION

Dynamic vibration absorbers (DVAs) are single degree of freedom (SDOF) passive mechanical systems that can be used to reduce the vibration of structures [1]. They are widely used to control vibrations in civil, aerospace and marine applications [2, 3]. These devices can operate to minimise the response of the hosting structure at a single frequency, which corresponds to the frequency of a tonal disturbance. In this case the device is designed to have
low internal damping and is usually called “vibration neutraliser” [4]. The principal limitation of this passive control system is that, if the device is slightly mistuned, it can fail in reducing the vibration and, in the worst case, can even increase the vibration level of the hosting structure. Semi-active vibration neutralisers can track changes in the operating conditions of the hosting structure and tune the resonance of the device to match the excitation frequency [2, 3, 5].

In presence of broadband excitation, DVAs can also be operated in such a way as to increase their internal damping. In this way it is possible to reduce the vibration of the hosting structure in a narrow frequency band around the absorber natural frequency. This type of device is usually called “tuned mass damper” or “dynamic vibration absorber”. Many criteria have been proposed for tuning the device natural frequency and internal damping in order to control a single mode of the structure [6]. This device can only be tuned to control the response of a selected mode of the structure and have negligible effects on the structural response at other frequencies. Several SDOF DVAs can be used to control more structural modes. Alternatively a multi degrees of freedom DVA can be used [7].

In this paper a new design of a tunable multi-modal vibration absorber is proposed. The absorber consists of a flexible beam of circular section and a cylindrical inertial mass mounted at its centre. The connection between the mass and the beam is realised by flexible fins which allow rotational vibration of the mass but do not allow relative displacement. When the two ends of the beam are attached to a vibrating structure, the device produces two control actions at low frequencies: a transverse force generated by the flexural vibration of the beam-mass system and a moment due to the torsional vibration of the inertial mass.

The device is equipped with a piezoelectric stack at the end of the beam. An applied voltage to the piezo actuator will tension the beam producing a change in its flexural natural frequencies. Moreover the inertial mass is equipped with two capacitors with one plate mounted on the beam and the other mounted on the inertial mass. When a voltage is applied across the capacitors a force between the two plates is generated and thus a change in the rotational natural frequency of the inertial mass is obtained.

The technical contribution of this paper is structured in three parts. In section 2 the design of the actuator is discussed including the actuation mechanism to tune the resonance frequencies of the DVA. In section 3 the mathematical model used to describe the dynamics of a panel subjected to broadband excitation and the control action produced by the DVA is briefly described. Finally simulation results are presented in section 4.

2. DESIGN OF A TUNING MULTI-MODAL VIBRATION ABSORBER

Figure 1 shows a scheme of the proposed multi-modal vibration absorber, which consists of a beam and a cylindrical mass $M$ connected via flexible fins.

![Figure 1: scheme of the multi-modal tuneable vibration absorber](image)
The flexible fins allow the rotation of the mass around the beam but are very stiff in the other directions thus do not allow transversal displacement between the mass and the beam. When the two ends of the absorber are attached to a vibrating structure, the absorber will generate a vertical force due to the flexural vibration of the beam-mass system and a moment due to the rotational vibration of the mass.

The $n$-th flexural natural frequency of a tensioned beam is equal to:

$$\omega_n = k_n \sqrt{\frac{1}{m} (EI k_n - F_a)}$$

where $k_n$ is the $n$-th bending wave number, which for a free-free beam is given in reference [10], $E$ is the young’s modulus, $I$ the moment of inertia of the beam cross section, $m$ is the density per unit length and $F_a$ is the applied axial force.

At one end, the beam is equipped with a piezoelectric stack actuator. When a tension $E_1$ is applied across the piezoelectric stack, the beam will be tensioned varying its flexural natural frequency according to equation (1). The two constitutive equations describing the electromechanical behavior of the piezo-actuator are given by [8]:

$$\begin{cases} 
V = T_{me} E_1 - 1/Z_m F_a \\
I_a = 1/Z_e E_1 - T_{me} F_a
\end{cases}$$

where $V$ is the velocity at the tip of the stack, $I_a$ is the current and $F_a$ is the force generated by the actuator. $T_{me}$ is the transduction coefficient given by:

$$T_{me} = \left. \frac{V}{E_1} \right|_{F_a = 0} = \left. \frac{I_a}{F_a} \right|_{E_1 = 0} = j \omega n d_{33}$$

where $\omega$ is the angular frequency in rad/sec, $j = \sqrt{-1}$ is the complex variable, $n$ is the number of laminas forming the stack and $d_{33}$ is the piezoelectric transduction coefficient. Also $Z_m$ is the short-circuit mechanical impedance given by:

$$Z_m = \left. \frac{F_a}{V} \right|_{E_1 = 0} = \frac{K_m}{j \omega n}$$

where $K_m$ is the axial stiffness of the lamina. Finally $Z_e$ is electrical impedance of the device given by:

$$Z_e = \left. \frac{E_1}{I_a} \right|_{F_a = 0} = \frac{1}{j \omega n C_e}$$

where $C_e$ is the capacitance of each piezoelectric lamina. Considering that the piezoelectric stack reacts off a rigid structure and considering that $V = K_b F_a$, where $K_b$ is the axial mobility of the beam, the force $F_a$ can be calculated from equations (2) as:

$$F_a = \frac{T_{me} Z_m}{K_b Z_m + 1} E_1$$

Equation (6) shows that a change in the applied voltage $E_1$ will produce a force on the beam along its axis and thus a change in the flexural natural frequency of the beam.

Moreover the inertial mass $M$ is equipped with two capacitors. One plate of each capacitor is attached to the beam and the other to the inertial mass as shown in figure 1. If a tension $E_2$ is applied to the capacitors a relative displacement $b$ between the plates leads to a force $F_c$ given by [9]:

$$F_c = - \frac{\varepsilon b E_2^2}{2d}$$

where $d$ is the distance between the capacitor plates and $\varepsilon$ is the permittivity. The equation of motion for the rotation of the inertial mass can be written as:

$$I_x \ddot{\theta} + c \dot{\theta} + k \theta - F_c = 0$$

where $c$ and $k$ are the damping and stiffness of the mounting fins, $I_x$ is the moment of inertia of the mass and $\theta$ is the angle of rotation. Assuming small rotations of the mass, $b$ can be
linearised as \( b = r \theta \) where \( r \) is the distance between the centre of rotation and the capacitor. Substituting equation (7) in (8) the rotation natural frequency of the inertial mass yields to:

\[
\omega_r = \sqrt{\frac{2dk + \varepsilon r_1 E_2^2}{2dl_x}} = \sqrt{\frac{k}{I_x}} \tag{9}
\]

Equation (9) shows that by changing the voltage across the capacitor plates the rotational natural frequency of the device can be varied. It is important to notice that the beam is assumed to be rigid in torsion and that a change in the rotational natural frequency of the device do not influence the flexural natural frequency of the beam mass system.

3. MATHEMATICAL MODEL

Figure 2 shows the free body diagram of a panel subjected to s broadband point forces and controlled by the multi modal absorber described in section 2. The DVA is attached to the panel at positions A and B in the centre of the panel along the x-axis. This position corresponds to the point of maximum displacement of the first mode and to a node (i.e. maximum rotation) of the second mode shape. Therefore this location is optimal for the controllability of the first two modes of the structure using the multi-modal absorber [10]. The cylindrical mass is attached at the centre of the beam at position C. In the experimental setup the device could be mounted on a relatively rigid support which can then be clamped to the structure under control. Another possibility is to attach the tips of the beam directly on a perforated plate.

Vector \( q_N = [w_N, \theta_x, \theta_y]^T \) is the column vector containing the velocity \( w_N \) in z direction and the angular velocities \( \theta_x, \theta_y \) around x and y axis and vector \( Q_N = [f_N, m_{xN}, m_{yN}]^T \) is the column vector containing the force \( f_N \) in z direction and the moments \( m_{xN}, m_{yN} \) around x and y axis at a generic location \( N \) of coordinate \( (x_N, y_N) \).

Considering the equilibrium of the forces acting on the panel, beam and mass and the compatibility conditions at the connection points A, B, and C, the following equations can be obtained:

\[
\begin{align*}
q_A &= Y_{AA} Q_A + Y_{BA} Q_B + Y_{PA} Q_P \\
q_B &= Y_{AB} Q_A + Y_{BB} Q_B + Y_{PB} Q_P \\
q_A &= -K_{AA} Q_A - K_{BA} Q_B + K_{CA} Q_C \\
q_B &= -K_{AB} Q_A - K_{BB} Q_B + K_{CB} Q_C \\
q_C &= -K_{AC} Q_A - K_{BC} Q_B + K_{CC} Q_C \\
q_C &= -H_w Q_C
\end{align*}
\]
where $Y_{NR}$ is the mobility matrix of a simply supported panel between an excitation vector $Q_N$ acting in a generic point $N$ and the velocity vector $q_R$ calculated at generic point $R$:

$$Y_{NR} = \begin{bmatrix} Y_{NwR} & Y_{mxNwR} & Y_{myNwR} \\ Y_{N\theta xR} & Y_{mxN\theta xR} & Y_{myN\theta xR} \\ Y_{N\theta yR} & Y_{mxN\theta yR} & Y_{myN\theta yR} \end{bmatrix} \tag{11}$$

Also $Y_{PR}$ represents the mobility matrix between the $s$ primary vertical forces acting on the panel and the velocity vector measured at location $R$ while $Q_P = [f_{p1} \ldots f_{ps}]^T$ is the column vector of the $s$ primary vertical forces.

$K_{NR}$ is the mobility matrix of a free-free beam between an excitation vector $Q_N$ acting in a point $N$ and the velocity vector $q_R$ measured on a point $R$:

$$K_{NR} = \begin{bmatrix} K_{fNwR} & 0 & K_{myNwR} \\ 0 & K_{myNwR} & 0 \\ K_{N\theta yR} & 0 & K_{myN\theta yR} \end{bmatrix} \tag{12}$$

The mobility expressions for the panel and beam can be found in reference [11]. Finally matrix $H_w$ is a mobility matrix of the inertial mass:

$$H_w = \begin{bmatrix} 1/(j\omega M) & 0 & 0 \\ 0 & j\omega I_x(\ddot{\theta} + j\omega c)/(j\omega c + \ddot{\omega}^2 \omega_x) & 0 \\ 0 & 0 & 1/(j\omega I_y) \end{bmatrix} \tag{13}$$

where $I_x$ and $I_y$ are the moment of inertia of the mass calculated along the $x$ and $y$ axis respectively. The simultaneous solution of equations (10) yields to:

$$X = G^{-1}P \tag{14}$$

where $X = [q_A, q_B, q_C, Q_A, Q_B, Q_C]^T$ is the column vector of the unknown and $G$ and $P$ are given by:

$$R = \begin{bmatrix} 1 & 0 & 0 & -Y_{AA} & -Y_{BA} & 0 \\ 0 & 1 & 0 & -Y_{AB} & -Y_{BB} & 0 \\ 1 & 0 & 0 & K_{AA} & K_{BA} & -K_{CA} \\ 0 & 1 & 0 & K_{AB} & K_{BB} & -K_{CB} \\ 0 & 0 & 1 & K_{AC} & K_{BC} & -K_{CC} \\ 0 & 0 & 1 & 0 & 0 & H_w \end{bmatrix} \quad P = \begin{bmatrix} Y_{PA}Q_P \\ Y_{PB}Q_P \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

where $I$ is a 3x3 identity matrix and $O$ is a 3x3 matrix of zeros.

4. SIMULATION RESULTS

In this preliminary study the dynamic of the beam has been approximated with a spring and a damper, thus the absorber has been approximated with a two degrees of freedom system consisting of a mass that oscillates linearly along the $z$-axis and angularly around the $x$-axis. Firstly, the linear and rotational actions of the absorber have been considered separately as schematically shown in figure 3.

![Figure 3](image)

Figure 3: scheme of the panel excited by 16 white point forces and controlled by (a) a single axial absorber and (b) a single rotational absorber both located in the centre
In the first case (a) the transverse force produced by the absorber is used to control the first flexural mode of the structure and thus its natural frequency equals the first resonance of the panel. In the second case (b) the moment produced by the absorber is used to control the second mode of the structure and thus its natural frequency equals the second resonance of the panel.

The physical and geometrical parameters of the panel and the inertial mass are summarised in table 1 and table 2 respectively.

Table 1: physical and geometrical parameters of the panel

<table>
<thead>
<tr>
<th>Panel parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions (l₁ × l₂)</td>
<td>0.464 x 0.314 m²</td>
</tr>
<tr>
<td>Thickness</td>
<td>1 x 10⁻³ m</td>
</tr>
<tr>
<td>Material</td>
<td>Aluminium ρ=2720 Kg/m³</td>
</tr>
<tr>
<td>Mass</td>
<td>Mₚ=0.4 Kg</td>
</tr>
<tr>
<td>Young Modulus</td>
<td>E=70 x 10⁹</td>
</tr>
<tr>
<td>Poisson’s coefficient</td>
<td>ν=0.33</td>
</tr>
<tr>
<td>Modal damping ratio</td>
<td>ζ=0.01</td>
</tr>
<tr>
<td>Boundary conditions</td>
<td>Simply supported</td>
</tr>
</tbody>
</table>

Figure 4 shows the kinetic energy of the panel before the axial absorber is mounted on the structure (faint line) and when the response of the first resonant mode is reduced using the axial absorber with low (dashed line) high (dash-dotted line) and an optimal (dotted line) value of the mechanical damping. When the mechanical damping is low the spectrum is sensibly reduced at the first resonance frequency but two new sharp peaks are visible in the spectrum. For very high values of mechanical damping the spectrum is shifted down in frequency. For an optimal value of the internal damping the structural response is reduced in a narrow frequency band around the first resonance frequency. Figure 5 shows the kinetic energy of the panel before the absorber is mounted (faint line), when the response of the second resonance mode is reduced using a rotational absorber with low (dashed line) high (dash-dotted line) and an optimal (dotted line) mechanical damping.

Table 2: physical and geometrical parameters of the inertial mass

<table>
<thead>
<tr>
<th>Sketch of the mass</th>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>h=0.02 m</td>
<td></td>
</tr>
<tr>
<td>External diameter</td>
<td>d₁=0.04 m</td>
<td></td>
</tr>
<tr>
<td>Internal diameter</td>
<td>d₂=0.02 m</td>
<td></td>
</tr>
<tr>
<td>Material density</td>
<td>ρ=2720 Kg/m³</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>0.05 Kg</td>
<td></td>
</tr>
<tr>
<td>Moments of inertia</td>
<td>Iₓ = 1.28 x 10⁻⁵ Kg m²</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Iᵧ = 1.28 x 10⁻⁵ Kg m²</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4: Kinetic energy of the panel without absorber (faint line), with the axial absorber when its mechanical damping is set to a very low (dashed line), to the optimal value (dotted line) and to a very high value (dash-dotted line).

Figure 5: Kinetic energy of the panel without absorber (faint line), with the rotational absorber when its mechanical damping is set to a very low (dashed line), to the optimal value (dotted line) and to a very high value (dash-dotted line).

Comparing figure 4 with 5 the rotational absorber shows a similar effect as that produced by the axial absorber at the first resonance frequency. When the mechanical damping is very low the response is reduced at the second resonance of the structure but two new sharp peaks are visible in the spectrum. For very high mechanical damping the mass results rigidly connected to the structure shifting down in frequency the second resonance of the panel. For an optimal value of the mechanical damping the response is reduced in narrow frequency band around the second resonance. Figure 6 shows the reduction in kinetic energy for different values of the damping ratio when the axial (solid line) and rotational (dashed line) absorbers are used to control the response of the panel integrated in the frequency band around the first and second resonance respectively. The graph shows that about 14 dB reduction can be achieved in the frequency band around the first resonance using an axial absorber and about 8 dB reduction can be obtained in the frequency band around the second resonance frequency using a rotational absorber. This result suggests that, combining the axial and rotational actions using an integrated absorber, significant reduction around the first and second resonances can be achieved with the same added weight to the structure.
The graph in figure 6 also shows that the gradient of the two curves is low around the minimum therefore a mistuning of the internal damping marginally affect the performance of the controller. For example a mistuning of 20% of the optimal internal damping causes an increase of less than 1 dB of the total kinetic energy from the optimal value. Therefore in this preliminary study the internal damping is fixed at the design stage and a mechanism for the online tuning is not considered.

Figure 7 shows the reduction in the kinetic energy of the panel integrated in the frequency band including the first two resonance frequencies with respect to the axial and rotational damping ratios. The graph shows that about 10 dB reduction can be achieved at low frequencies, when the flexural vibration of the panel is controlled by the integrated absorber.

![Figure 6: Reduction of the kinetic energy using an axial absorber integrated in the 8-60 Hz frequency band (solid line) and using a rotational absorber integrated in the 45-95 Hz frequency band as function of the damping ratio of the absorber.](image)

![Figure 7: reduction of the kinetic energy of the panel integrated in the frequency band 10-95 Hz using an integrated absorber as function of the rotational and axial damping ratios of the absorber.](image)

5. CONCLUSIONS

In this paper the design of a new multi-modal vibration absorber has been discussed. The absorber consists of a beam with a cylindrical inertial mass attached at its centre with flexible fines which only allows rotational vibration of the inertial mass around the beam axis. The beam is supported from the two ends. The transverse vibration of the beam-mass system and the rotational vibration of the inertial mass produce at low frequency two actions on the structure where the absorber is attached: a vertical control force and a control moment. The
absorber can therefore be used to control the response of two resonant modes of a flexible structure if the two resonances of the absorber matches those of the structure. Therefore the absorber has been equipped with a piezoelectric stack actuator which can generate an axial force along the beam when a voltage across the actuator is applied. This will change the flexural resonance frequencies of the beam. Moreover the absorber is equipped with two capacitors with one plate mounted on the inertial mass and the other on the beam. When a voltage is applied across the capacitors a relative displacement of the plates will generate a force and thus will produce a change in the rotational natural frequency of the device. This design choice will in principle guarantee that a change in one of the two resonance frequency of the device will have no effect on the other making the tuning operation simpler. The absorber can therefore control two modes of the structure reacting off one mass. Moreover the absorber can be modified in such a way as to produce two control moment effects along two orthogonal axis.

REFERENCES


