NUMERICAL STUDY OF A SUBMERGED HULL WITH INTERNAL MASSES

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ABSTRACT

The onboard machinery of a marine vessel include the engines, generators, main motor, gearboxes and auxiliary equipment, and accounts for a large amount of the total mass of the vessel. In the medium to high frequency ranges, the hull of the vessel, being a flexible structure, has high modal density and short wavelengths and is more appropriately modelled using Statistical Energy Analysis (SEA). However, the onboard machinery are rigid body components with low modal density and well defined resonances, and are more suitably modelled using a deterministic approach such as finite element analysis (FEA). This work presents the dynamic responses of a simplified physical model of a submarine pressure hull, using both deterministic and statistical numerical models. A finite element / boundary element (FE/BE) model of a fluid loaded cylindrical shell with an internal floor-mass system is initially developed. A hybrid finite element / Statistical Energy Analysis (FE/SEA) model of the same structure is then developed. Two excitation cases are considered. In the first case, forces are applied externally to the cylindrical shell to simulate excitation from the propeller. In the second case, forces are applied to the internal mass to simulate excitation from the onboard machinery. For the two excitation cases, the vibrational energy levels of the shell are compared.

1. INTRODUCTION

The dynamic response of cylindrical shells is of interest in many engineering applications, particularly in the aerospace and maritime industries, as a cylinder represents the fundamental shape of an aircraft fuselage or submarine pressure hull. Such structures are generally partitioned with internal floor structures mounted with machinery.

Peterson and Boyd [1] used the Rayleigh-Ritz method to study the free vibration of a cylindrical shell with internal floor. The effects of vertical position and thickness of the floor
on the natural frequencies and mode shapes were examined. Langley [2] studied the free vibration of a simply supported stiffened cylindrical structure using the dynamic stiffness technique. Missaoui et al. [3] studied the free and force vibration of a cylindrical shell with a floor partition. The structural coupling was simulated using artificial spring systems. Their work was then extended to examine the contributions of the structural modes to the internal sound pressure [4]. Lee and Choi [5] used the receptance method to calculate the dynamic characteristics of a simply supported cylindrical shell with an interior rectangular plate, and compared their results with experimental modal test results. In ships and submarines, the internal structures and machinery are often resiliently mounted on the hull for the purpose of vibration isolation. The complexity of structure makes it impractical to examine all structural details even with experimental or numerical approaches. To capture the essential features of the problem, the internal structure can be simplified as a spring-mass system. Guo [6] showed that the effect of an internal spring-mass system of a cylindrical shell alters the sound scattering at low to medium frequencies.

Submarine hulls are generally subjected to various loads ranging from lower frequency sources such as the propulsion system to higher frequency sources such as the on-board machinery. The lower frequency sources will tend to excite the global modes of the pressure hull, while the higher frequency sources will tend to excite the local modes in which vibration occurs predominantly in a small region of the structure. At high frequencies, the structural wavelengths become very small thus requiring very small element sizes in a finite element or boundary element model. Such deterministic methods are generally limited to the low frequency range. The Statistical Energy Analysis (SEA) method has been applied successfully to solve high frequency problems in the marine, aerospace and automotive industries. The validity of the SEA equations is usually limited to high frequencies because of the underlying assumptions of high modal density and weak coupling between structural subsystems. In the mid-frequency range, the dynamic behaviour of a structure is the combination of long wavelength global modes and short wavelength local modes. An emerging approach to solve mid-frequency problems is the Hybrid FE-SEA method [7, 8] that combines the finite element method with statistical energy analysis.

In this paper, a fully coupled finite element / boundary element (FE/BE) model of a fluid loaded cylindrical shell closed at each end by circular plates and with an internal floor-mass system is developed. The onboard machinery is modelled as a discrete internal mass. A hybrid FE-SEA model of the structure is also developed. Two excitation cases are considered corresponding to external excitation of the shell and internal excitation of the mass. The effects of both the internal floor-mass system and the two load cases on the shell vibrational responses at low to medium frequencies are examined.

2. NUMERICAL FORMULATION

The numerical FE/BE model is developed as follows. Finite elements are used to represent the structure and boundary elements are used to represent the unbounded surrounding fluid medium. A fully coupled FE/BE model is achieved by the continuity of the normal velocity on the wetted surface and the acoustic pressure acting normally on the surface of structure. Assuming a time harmonic dependence of the form $e^{j\omega t}$, the dynamic equilibrium equation for an elastic structure is given by [9]

$$M\ddot{q} + C\dot{q} + Kq = F$$  \hspace{1cm} (1)

where $M$ is the global mass matrix, $C$ is the global damping matrix and $K$ is the global stiffness matrix. $\ddot{q}$, $\dot{q}$ and $q$ are the nodal acceleration, velocity and displacement vectors, respectively. $F$ is the external force vector. Using the mode superposition principle, the finite element nodal point displacements can be obtained as
\[ U(t) = \sum_{i=1}^{m} \Phi_i x_i(t) \]  

where \( U \) is the vector of nodal point displacements, \( \Phi_i \) is the \( i^{th} \) modeshape vector, \( x_i \) is the \( i^{th} \) mode displacement, \( m \) is the total number of modes, and \( t \) is the time variable.

The acoustic wave equation based on the Helmholtz equation is given by [10]

\[ \frac{1}{c_f^2} \ddot{p}_f + k_f^2 p_f = 0 \]  

where \( k_f = \omega / c_f \) is the wavenumber of the fluid, \( \omega \) is the angular frequency, and \( c_f \) is the speed of sound in fluid. \( p_f \) is the complex sound pressure and \( \ddot{p}_f \) denotes the second derivative with respect to time. Using the Neumann boundary condition, the pressure gradient normal to the boundary \( \Gamma_f \) is given by

\[ \frac{\partial p_f}{\partial n} = \rho_f \omega^2 \dot{u}_f \]  

where \( \rho_f \) is the density of the fluid, \( \dot{u}_f \) is the velocity of a fluid particle normal to the surface, and \( n \) is the unit outward normal at the surface point.

Applying Green’s second theorem to equation (3), the Helmholtz boundary integral equation can be derived to transform the three-dimensional problem into a two-dimensional formulation on the boundary. The Helmholtz integral equation can take the form as [11]

\[ c_r p_f(r) = \int_{\Gamma_f} \left( p_f(r) \frac{\partial G(|r - r_0|)}{\partial n} - G(|r - r_0|) \frac{\partial p_f}{\partial n} \right) \, d\Gamma_f \]  

\[ c_r = \begin{cases} 
 1 & r \in \Omega_{ext} \\
 0.5 & r \in \Gamma_f \\
 0 & r \in \Omega_{int} 
\end{cases} \]

where \( c_r \) is the Green’s constant depending on the location of \( r \) in the fluid domain, and \( r_0 \) represents a point located on the boundary \( \Gamma_f \). Based on this formulation, the pressure anywhere in the acoustic domain can be determined once the pressure and normal fluid velocity on the boundary are known. \( G(|r - r_0|) \) denotes the free space Green’s function which is given by

\[ G(|r - r_0|) = \frac{e^{-ik_j |r - r_0|}}{4\pi |r - r_0|} \]  

Figure 1 shows the fluid domain divided by the boundary \( \Gamma_f \) into an exterior fluid domain \( \Omega_{ext} \) and an interior fluid domain \( \Omega_{int} \).
Figure 1. Exterior and interior acoustic domain showing the boundary and the normal vectors

Discretization of the boundary integral equation can be presented using a collocation boundary element formulation yielding the following linear boundary element system equation [12]

$$\mathbf{H} \mathbf{p}_f = \mathbf{G}_f \omega^2 \mathbf{u}_f$$

(7)

$\mathbf{H}$ and $\mathbf{G}$ are the frequency dependent boundary element influence matrices.

The resulting system of equations for the submerged structure is given by [13]

$$\begin{bmatrix} \mathbf{K} - i\omega \mathbf{C} - \omega^2 \mathbf{M} & -\mathbf{C}_{sf} \\ -i\omega \mathbf{C} \mathbf{f}_s & \mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{u}_s \\ \mathbf{p}_f \end{bmatrix} = \begin{bmatrix} \mathbf{f}_s \\ \mathbf{f}_f \end{bmatrix}$$

(8)

where $\mathbf{f}_s$ and $\mathbf{f}_f$ are respectively the nodal structural forces and forces due to fluid loading acting on the surface of structure. $\mathbf{u}_s$ and $\mathbf{p}_f$ are nodal displacement and nodal pressure vectors, respectively. $\mathbf{C}_{fs}$ and $\mathbf{C}_{sf}$ are the coupling matrices corresponding to the two coupling conditions: (i) equilibrium of the acoustic pressure and normal stress at the wetted interface, and (ii) continuity of the fluid particles and structural nodal velocity normal to the wetted surface, that is [12,13]

$$\mathbf{f}_f = \mathbf{C}_{sf} \mathbf{p}_f$$

(9)

$$\mathbf{u}_f = i\omega \mathbf{C}_{fs} \mathbf{u}_s$$

(10)

3. **NUMERICAL MODEL OF CYLINDRICAL SHELL WITH INTERNAL MASS**

The floor plate is coupled with the cylindrical shell at a position centred vertically and a solid rectangular mass is located centrally on the floor, as shown in Figure 2. The dimensions and material properties of the model are listed in Table 1.

Figure 2. Schematic diagram of the cylindrical shell with an internal mass-floor structure
<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Physical dimensions and values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell radius</td>
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<td>m</td>
</tr>
<tr>
<td>Shell thickness</td>
<td>0.04</td>
<td>m</td>
</tr>
<tr>
<td>Shell length</td>
<td>45</td>
<td>m</td>
</tr>
<tr>
<td>Circular Plate radius</td>
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<td>m</td>
</tr>
<tr>
<td>Circular Plate thickness</td>
<td>0.04</td>
<td>m</td>
</tr>
<tr>
<td>Internal floor</td>
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</tr>
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<td>Internal floor thickness</td>
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<td>m</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
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<td>Pa</td>
</tr>
<tr>
<td>Mass density of steel</td>
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<td>kg/m³</td>
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<tr>
<td>Poisson’s ratio</td>
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<tr>
<td>Structural loss factor</td>
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<tr>
<td>Lumped mass dimensions</td>
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<td>m</td>
</tr>
<tr>
<td>Lumped mass density</td>
<td>7860</td>
<td>kg/m³</td>
</tr>
</tbody>
</table>

Table 1: Physical parameters for the cylindrical shell with an internal mass-floor structure

### 3.1 FE/BE model

MSC/Nastran [14] was used to create the finite element model which was then imported into VA-One software [15]. In the finite element model, Quad8 shell elements were used to model the cylindrical shell and circular end plates. Quad8 shell elements were also used to model the interior rectangular floor plate. Hex8 elements were used to model the mass block. The FE mesh of the cylindrical shell and internal floor/mass is shown in Figure 3.

![Finite element mesh of the cylindrical shell with an internal mass-floor structure](image)

Figure 3. Finite element mesh of the cylindrical shell with an internal mass-floor structure

The boundary element model is extracted from the finite element model given in Figure 3 using the BE solver in VA-One. The BE fluid is connected to the outer surface (wetted surface) of the entire structure. The boundary element model consists of 5768 triangular boundary elements and 7012 fluid nodes. The detailed boundary element mesh is shown in Figure 4.

Two excitation cases are modelled. In the first case, a radial point force is applied externally to the cylindrical shell. In the second case, a point force is applied to the internal mass. The locations of the excitation forces are shown in Figure 5.
3.2 Hybrid FE-SEA model

In the hybrid FE-SEA method, the structure is represented as an assembly of deterministic FE components known as the master system and SEA subsystems with random properties. In SEA, the system is modelled as an assembly of subsystems. The power balance equation of each subsystem is given by [16]

$$P_i = \omega \eta_i E_i + \omega \sum_{j=1}^{N} \eta_{ij} n_i \left( \frac{E_i}{n_i} - \frac{E_j}{n_j} \right)$$

where $P_i$, $E_i$, $n_i$, and $\eta_i$ are respectively the power input, vibrational energy, modal density, and loss factor of subsystem $i$, and $\eta_{ij}$ are the coupling loss factors. To couple the FE and SEA methods, the hybrid FE-SEA equation can be derived as [8]

$$P_i + P_{\text{ext}} = \omega (\eta_i + \eta_{d,i}) E_i + \omega \sum_{j=1}^{N} \eta_{ij} n_i \left( \frac{E_i}{n_i} - \frac{E_j}{n_j} \right)$$

Compared with the standard SEA equation given by equation (11), the hybrid FE-SEA equation has two additional terms: (i) a contribution $P_{\text{ext}}$ to the input power arising from forces applied directly to the master system; (ii) the additional loss factor $\eta_{d,i}$ of the master
system. These two additional terms can be expressed analytically as a function of the total dynamic stiffness matrix and the cross-spectral matrix of the external load applied to the master system.

The cylindrical shell with an internal mass-floor structure was modelled using the Hybrid FE-SEA module in the VA-One software, as shown in Figure 6. The mass block was modelled as an FE subsystem while the cylindrical shell, circular end plates and the floor were modelled as SEA subsystems. The SEA junctions are represented by red lines and hybrid FE-SEA junctions with blue lines. The water was modelled as a semi-infinite-fluid (SIF) subsystem. The fluid load was applied to the outer surface of the structure by connecting the SIF subsystem to the SEA subsystems of cylindrical shell and circular end plates.

![Hybrid FE-SEA model of the cylindrical shell with an internal mass-floor structure and surrounding water](image)

Figure 6. Hybrid FE-SEA model of the cylindrical shell with an internal mass-floor structure and surrounding water

4. RESULTS AND DISCUSSION

In what follows, the numerical calculations are carried out using the physical parameters listed in Table 1. The cylindrical shell and internal floor are assumed to have the same thickness and material properties. The speed of sound in water is 1500 m/s and the density of water is 1000 kg/m³. In both the coupled FE/BE model and the hybrid FE-SEA model, the structural damping loss factors are set as 0.01.

4.1 Frequency responses

In order to investigate the influence of the internal floor-mass system on the structural response of the cylindrical shell, the FE/BE results for the axial and radial displacements at the edge of cylindrical shell are presented in Figures 7-10, with and without the internal floor-mass system and for the two excitation cases. In Figures 7 and 8, the excitation force is applied externally at the edge of the cylindrical shell, as shown in Figure 5(a). In Figures 9 and 10, the excitation force is applied internally at the corner of the lumped mass, as shown in Figure 5(b). The frequency responses were predicted at the edge of the cylindrical shell.

In Figures 7 and 8, it can be observed that the presence of the internal floor-mass system has very little effect at low frequencies for the case of external excitation of the shell. The peaks in the frequency responses correspond to the bending circumferential modes of the cylindrical shell. In contrast, for the case of internal excitation of the mass (Figures 9 and 10), the floor-mass system significantly affects the frequency responses, in which the peaks are
now a combination of bending circumferential modes of the shell, breathing (axisymmetric) circumferential modes of the shell and bending modes of the floor-mass system. Operational deflection shapes of the cylindrical shell with an internal floor-mass system at frequencies of 21 Hz and 78 Hz are shown in Figures 11 and 12, respectively. In Figure 11, the system is under external excitation of the shell. It is observed that the global shell bending mode is unaffected by the internal structure. In Figure 12, bending resonance of the internal lumped mass under excitation is observed.

![Figure 7](image1.png)

Figure 7. Magnitude of axial displacement of the cylindrical shell with and without an internal floor-mass system, due to external excitation of the shell

![Figure 8](image2.png)

Figure 8. Magnitude of radial displacement of the cylindrical shell with and without an internal floor-mass system, due to external excitation of the shell
Figure 9. Magnitude of axial displacement of the cylindrical shell with and without an internal floor-mass system, due to internal excitation of the internal mass

Figure 10. Magnitude of radial displacement of the cylindrical shell with and without an internal floor-mass system, due to internal excitation of the internal mass
Figure 11. Operational deflection shape of the cylindrical shell with an internal floor-mass system due to external excitation of the shell, at 21 Hz

Figure 12. Operational deflection shape of the cylindrical shell with an internal floor-mass system due to internal excitation of the internal mass, at 78 Hz

4.2 Comparison of FE/BE and Hybrid FE-SEA results

In order to compare with results from the hybrid FE-SEA model, the energy response of the structure is also computed using the fully coupled FE/BE model. The total energy is the summation of potential energy and kinetic energy of the vibrating structure. Since the potential energy equals to the kinetic energy of the system at resonance, the total energy is twice the kinetic energy [15], as follows

\[ E = 2E_k = \frac{1}{2} V^H M V \]  

where \( V \) is the nodal velocity vector of the modal response, \( M \) is the mass matrix, \( E \) and \( E_k \) are respectively the total and kinetic energies of the structure, and ‘\( H \)’ denotes the Hermitian operator. In VA-One, the modal responses of structures are described by three wave fields: out of plane bending, in-plane extension and shear waves.

In general, the curvature of the cylindrical shell significantly affects the wave propagation below the ring frequency. The ring frequency of a cylindrical shell is the frequency at which the wave length of longitudinal waves equals the mean circumference of the cylindrical shell, and is given by [17]

\[ f_r = \frac{c_L}{2\pi a} \]  

where \( a \) is the mean radius of the cylindrical shell, \( c_L = \sqrt{E/\rho(1-\mu^2)} \) is the longitudinal wave velocity, \( E \) is the Young’s modulus, \( \mu \) is Poisson’s ratio, and \( \rho \) is the density of the
shell. Substituting the physical parameters of the cylindrical shell into equation (14) results in a ring frequency of 266 Hz.

Comparison of the deterministic and hybrid approaches were conducted in the frequency range up to 300 Hz. Figures 13 and 14 present the energy levels of the cylindrical shell with the internal floor-mass system from both the FE/BE model and the Hybrid FE/SEA model, for external excitation of the shell (Figure 13) and internal excitation of the mass (Figure 14).

![Figure 13. Energy levels of the cylindrical shell with an internal floor-mass system, due to external excitation of the shell](image13.png)

![Figure 14. Energy levels of the cylindrical shell with an internal floor-mass system, due to internal excitation of the internal mass](image14.png)

A better convergence can be observed in the energy levels obtained from the deterministic FE/BE model and the Hybrid FE-SEA model in Figure 14. This is attributed to the greater number of local modes excited by the internal excitation of the floor-mass system, resulting in
more reliability in the FE-SEA model. In Figure 13, as only global modes of the cylindrical shell are predominantly excited by the external force, there is insufficient modal density for the Hybrid FE-SEA model. Figure 15 presents the modal density of the cylindrical shell for the radial and axial modes, and shows the ring frequency at 266 Hz. In Figures 13 and 14, a slight increase in the energy levels in the Hybrid FE-SEA results at the ring frequency can be observed.

![Modal density of the cylindrical shell](image)

Figure 15. Modal density of the cylindrical shell

5. CONCLUSIONS

The structural responses of a submerged finite cylindrical shell with an internal mass have been presented using both deterministic and statistical modelling techniques. A numerical FE/BE model of the cylindrical shell with an internal floor-mass system, which represents a longitudinal floor mounted with machinery in a submerged pressure hull, was developed. At low frequencies, the dynamic responses of the fluid loaded shell with and without an internal structure were compared. For external excitation of the cylinder, the influence of the floor-mass system was very small and the frequency responses are dominated by the bending circumferential shell modes. For excitation of the internal mass, the floor-mass dynamics dominate the shell dynamic responses. At higher frequencies, the energy responses obtained from a hybrid FE/SEA model have been presented. Good agreement between the spatially averaged energy levels of the cylindrical shell with an internal mass from the FE/BE model and results from the hybrid FE/SEA model were obtained for the case of internal excitation of the mass, in which a greater number of local modes were excited.

REFERENCES


