THE VERTICAL SEISMIC ISOLATOR BY USING A LINK-CRANK MECHANISM

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ABSTRACT

In recent years, various seismic isolators have been developed to prevent earthquake damage to valuable art. Many seismic isolators only enable horizontal motion because horizontal vibration is the usual cause of falling objects. However, development of a seismic isolator designed for vertical vibration is necessary, since great vertical vibration earthquakes, such as the 2004 Niigata Prefecture Chuetsu Earthquake, have occurred. Vertical seismic isolators have been developed in the past, but some of their characteristics, such as the large size of the static deflection of the spring and the increased height of the seismic isolator, are undesirable. In this study, we developed a vertical seismic isolator that can support loads using a horizontal spring without requiring a vertical spring. Thus, our vertical seismic isolator has a major stroke and lower installation height. We verified this vertical seismic isolator’s performance through shaking tests and mathematical calculation. The calculation verified the seismic isolation because the average reduction rate of acceleration in six seismic waves was 0.32. Sinusoidal waves of 2, 3, and 4 Hz were inputted to the base plate of the seismic isolator and, as a result of the response of the top plate, the effect of seismic isolation was verified in the experimental model because the average reduction rate of acceleration was 0.57. In addition, we uncovered some problems for this vertical seismic isolator’s practical use.

1. INTRODUCTION

In recent years, to prevent art objects and valuable, accurate instruments from falling and sustaining damage during an earthquake, various provisions of seismic isolation have been developed. The cost to prepare for seismic isolation, however, is high [1]. In addition, although seismic isolators for small objects have been put into practical use, many seismic isolators are only efficient for horizontal motion because horizontal vibration is most often the cause of falling objects [2, 3]. However, development of seismic isolators that are effective for vertical vibration is necessary to cope with great vertical-vibration earthquakes, such as the 2004 Niigata Prefecture Chuetsu Earthquake. Previous studies have developed vertical
seismic isolators that combine a vertical spring and link mechanism to maintain the horizontal while using a negative stiffness mechanism to reduce natural frequency [4, 5]. However, for the vertical spring to absorb weight, the static deflection of the spring must increase when the natural frequency decreases, resulting in buckling of the vertical spring. Additionally, using a negative stiffness in order to reduce the spring’s static deflection increases the height of the seismic isolator and, since a link mechanism must be added to the seismic isolator, the stroke must be increased when the natural frequency is reduced. It is therefore difficult to reduce natural frequency and downsize the seismic isolator.

In this study, using a mechanism that combines a link and crank, and downsizing the seismic isolator by reducing the amount of expansion and contraction with a stiff spring, we propose a vertical seismic isolator that can reduce natural frequency with apparent spring constant. The mechanism of this vertical seismic isolator can support a vertical load using only a horizontal spring without using a vertical spring. This mechanism can increase the stroke of the seismic isolator and reduce its height. We verify the performance of this seismic isolator by experiment and mathematical calculation.

2. VERTICAL SEISMIC ISOLATOR

2.1 Mechanism of the vertical seismic isolator

The experimental device of the vertical seismic isolator is shown in Figure 1 and its schematic diagram is shown in Figure 2. This link-crank seismic-isolation mechanism consists of a compound link mechanism, which adopts a crankshaft of the tilt link and a parallel link mechanism. A spring force acts in an up-and-down manner though the link, providing a spring force parallel to the shaft though the connecting rod. The top plate and base plate, which are fixed on the link, maintain a horizontal position because the center shaft and the link retain a parallelogram shape. Additionally, the initial tension may be adjusted to correspond to the mass of physical objects because both ends of the horizontal spring are joined to a bolt that can adjust the spring’s expansion and contraction. The damping is obtained by friction arising in an axis of rotation.

Figure 1. The proposed vertical seismic isolator with a link and crank.

Figure 2. Schematic diagram of the proposed vertical seismic isolator.
2.2 Link-crank mechanism

Figure 3 shows a simplified figure of the link, crank, and connecting rod. In Figure 3, \( l_a \) is the link length, \( l_b \) is the crank length, \( l_c \) is the connecting rod length, \( x_d \) is the displacement of the spring head, \( \theta_a \) is the link angle in the equilibrium condition, \( \theta_b \) is the crank angle, and \( \theta_c \) is the rod angle. When \( l_d \) is the distance from the link hinge to the spring head, \( l_d \) is

\[
\begin{equation}
\begin{aligned}
l_d &= l_b \cos \theta_b + l_c \cos \theta_c
\end{aligned}
\end{equation}
\]

Then, \( l_{d0} \) is the \( l_d \) in the equilibrium condition and spring displacement \( x_d \) is

\[
\begin{equation}
\begin{aligned}
x_d &= l_d - l_{d0}
\end{aligned}
\end{equation}
\]

Additionally, when the rod hinge has an offset \( h \), crank angle \( \theta_b \) and rod angle \( \theta_c \) is, from \( l_b \sin \theta_b = l_c \sin \theta_c + h \),

\[
\begin{equation}
\begin{aligned}
\theta_c &= \sin^{-1} \left( \frac{l_b \sin \theta_b - h}{l_c} \right)
\end{aligned}
\end{equation}
\]

\( f_h \) is the horizontal spring force and \( f_c \) is the force acting on the rod. Torque \( T_b \) acting on the crank is, from \( f_h = f_c \cos \theta_c \),

\[
\begin{equation}
\begin{aligned}
T_b &= f_h l_b \sin(\theta_b + \theta_c) = f_c l_b \frac{\sin(\theta_b + \theta_c)}{\cos \theta_c}
\end{aligned}
\end{equation}
\]

In order to support a seismic isolator load \( f \) on the link, the necessary torque \( T_a \) is

\[
\begin{equation}
\begin{aligned}
T_a &= 2l_a f \cos \theta_a
\end{aligned}
\end{equation}
\]

Since the torque \( T_a \) acting on the link is equivalent to the torque \( T_b \) acting on the crank, from Eq. (4) = Eq. (5),

\[
\begin{equation}
\begin{aligned}
f &= f_h \frac{l_b \sin(\theta_b + \theta_c)}{2l_a \cos \theta_a \cos \theta_c}
\end{aligned}
\end{equation}
\]

By using Eq. (6), the compression rate of link and crank \( \beta \) is written as Eq. (7).

\[
\begin{equation}
\begin{aligned}
\beta &= \frac{l_b \sin(\theta_b + \theta_c)}{2l_a \cos \theta_a \cos \theta_c}
\end{aligned}
\end{equation}
\]

Furthermore, the relation between a displacement \( x_a \) of the seismic isolator from the equilibrium condition and a spring displacement \( x_d \) is \( x_a = \beta^2 x_d \). Thus, when a seismic isolator displacement is \( x_a \), potential energy is stored in the spring constant of spring \( k \) as Eq. (8).

\[
\begin{equation}
\begin{aligned}
\frac{1}{2} k x_a^2 &= \frac{1}{2} k \beta^2 x_d^2
\end{aligned}
\end{equation}
\]

In fact, the relation of \( k \) and an apparent spring constant \( k_a \) is written as below:

\[
\begin{equation}
\begin{aligned}
k_a &= \beta^2 k
\end{aligned}
\end{equation}
\]
In addition, link angle \( \theta_a \) is changed to \( \theta \), and crank angle \( \theta_b \) and rod angle \( \theta_c \) are written as below by using \( \theta \):

\[
\theta_b = \theta + \alpha, \quad \theta_c = \sin^{-1}\left(\frac{l_b}{l_c} \sin(\theta + \alpha) - \frac{h}{l_c}\right)
\]

(10)

In this regard, \( \alpha \) is fixed angle. In fact, the compression rate \( \beta \) is,

\[
\beta = \frac{l_b \sin(\theta + \alpha + \sin^{-1}(\frac{l_b}{l_c} \sin(\theta + \alpha) - \frac{h}{l_c})))}{2l_a \cos \theta \cos(\sin^{-1}(\frac{l_b}{l_c} \sin(\theta + \alpha) - \frac{h}{l_c}))}
\]

(11)

Figure 3. Simplified figure of link-crank mechanism.

In this way, the system’s natural frequency is reduced by using a stiff spring, and the seismic isolator can be downsized because the spring’s expansion and contraction are reduced. Moreover, an apparent spring constant can be changed by adjusting offset \( h \), since an offset on a force application unit can be arbitrarily adjusted. Figure 4 shows the restitution force on horizontal axis \( z \) where the gradient in Figure 4 is \( k_a \).

Figure 4. The restitution force depending on relative displacement.
2.3 Dynamic model and motion equation

Figure 5 shows the dynamic model of the vertical seismic isolator in this study. The mass of a physical object of seismic isolation is \( M \). The link-crank mechanism expresses in mass point on the links and this mass is \( m \). The link length is \( l/2 \), the apparent spring constant is \( k_a \), the link angle is \( \theta \), the displacement from the equilibrium condition is \( z_0 \) on the base plate, the displacement from the equilibrium condition is \( z_1 \) on the top plate, the vertical displacement of the link-crank mechanism is \( z_2 \), and the horizontal displacement of it is \( x \). In this regard, the mass and rotary inertia are neglected on the link and \( \theta \) is the plus direction when a relative displacement \( z \) increases. The internal damping of the device is considered by replacing a dash pot and the damping constant is \( c \). Additionally, although \( k_a \) is a function of \( \theta \), \( k_a \) is considered as a constant below calculation because it is nearly linear in shape. The relations between \( z_0, z_1, z_2, \) and \( z \) are [6]

\[
 z_1 = z + z_0, \quad z_2 = \frac{1}{2}(z + z_0) \quad (12), (13)
\]

By using link angle \( \theta \) and link length \( l/2 \), relative displacements \( z \) and \( x \) are,

\[
 z = \frac{l}{2} \sin \theta \quad (14)
\]

\[
 x = \frac{l}{2} \cos \theta \quad (15)
\]

By substituting Eq. (14) for Eq. (12) and Eq. (13),

\[
 z_1 = z_0 + \frac{l}{2} \sin \theta \quad (16)
\]

\[
 z_2 = z_0 + \frac{1}{2} \sin \theta \quad (17)
\]

Moreover, by differentiating Eqs. (15)-(17) with respect to the variable of time,

\[
 \dot{z}_1 = \dot{z}_0 + \frac{l}{2} \dot{\theta} \cos \theta \quad (18)
\]

\[
 \dot{z}_2 = \frac{1}{2}(\dot{z}_1 + \dot{z}_0) = \dot{z}_0 + \frac{l}{2} \dot{\theta} \cos \theta \quad (19)
\]

\[
 \ddot{x} = -\frac{l}{2} \dot{\theta} \sin \theta \quad (20)
\]

By obtaining the kinetic energy \( T \) of this device from Eq. (18)-(20),

\[
 T = \frac{1}{2} M \dot{z}_1^2 + \frac{1}{2} m \dot{z}_2^2 + \frac{1}{2} m \dot{x}^2
 \]

\[
 = \frac{1}{2} M (\dot{z}_0 + \frac{l}{2} \dot{\theta} \cos \theta)^2 + \frac{1}{2} m(\dot{z}_0 + \frac{l}{2} \dot{\theta} \cos \theta)^2 + \frac{1}{2} m(-\frac{1}{2} \dot{\theta} \sin \theta)^2
 \]

(21)

Secondly, by obtaining the potential energy \( U \) of this device from Eq. (14),
\[ U = \frac{1}{2} k_a z^2 = \frac{1}{2} k_a l^2 \sin^2 \theta \] (22)

Since Eq. (21) and (22) are obtained, by determining Lagrangian \( L \),

\[
L = T - U = \frac{1}{2} M (\ddot{z}_0 + l\dot{\theta} \cos \theta)^2 + \frac{1}{2} m (\ddot{z}_0 + \frac{l}{2} \dot{\theta} \cos \theta)^2 + \frac{1}{2} m (\frac{l}{2} \dot{\theta} \sin \theta)^2 - \frac{1}{2} k_a l^2 \sin^2 \theta
\] (23)

By obtaining the dissipation function \( F \),

\[ F = \frac{1}{2} c \dot{z}^2 = \frac{1}{2} c l^2 \dot{\theta}^2 \cos^2 \theta \] (24)

In order to obtain the motion equation, by substituting Eq. (23) and (24) for the Lagrange equation

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} + \frac{\partial F}{\partial \dot{\theta}} = 0
\]

and rearranging this equation,

\[
(M \cos \theta + \frac{m}{4 \cos \theta}) l \ddot{\theta} - M l \dot{\theta} \sin \theta + k_a l \sin \theta + c l \dot{\theta} \cos \theta = -(M + \frac{m}{2}) \ddot{z}_0
\] (25)

In this way, motion equation is obtained. Eq.(25) is the motion equation of this vertical seismic isolator. Here, \( \dot{\theta}^2 \) is neglected because it is a minute term,

\[
(M \cos \theta + \frac{m}{4 \cos \theta}) l \ddot{\theta} + k_a l \sin \theta + c l \dot{\theta} \cos \theta = -(M + \frac{m}{2}) \ddot{z}_0
\] (26)

Eq. (26) is a simplified motion equation of this seismic isolator. Eq. (26) is used in the mathematical calculation, as described below.

Figure 5. Analytical model of the vertical seismic isolator.
3. MATHEMATICAL CALCULATION AND EXPERIMENT

3.1 Mathematical calculation of characteristic frequency

In assigning forced displacement with the base plate, we verified the frequency characteristic of the vertical seismic isolator by inputting a sinusoidal wave of acceleration $a_0$. Table 1 details the specifications using this mathematical calculation, which uses the Runge-Kutta method. In this regard, $f_n$ is the calculated frequency characteristic of the vertical seismic isolator. Although $c$ can be found by using a half-bandwidth method from the experimental frequency response, $c$ is only a reference value because the experiment cannot be run in low-frequency vibration. The reason is described in Section 3.4.

<table>
<thead>
<tr>
<th>Material Parameter</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>2.00 kg</td>
</tr>
<tr>
<td>$m$</td>
<td>0.91 kg</td>
</tr>
<tr>
<td>$k$</td>
<td>3658 N/m</td>
</tr>
<tr>
<td>$l_a$</td>
<td>0.12 m</td>
</tr>
<tr>
<td>$l_b$</td>
<td>0.023 m</td>
</tr>
<tr>
<td>$l_c$</td>
<td>0.07 m</td>
</tr>
<tr>
<td>$h$</td>
<td>0.011m</td>
</tr>
<tr>
<td>$c$</td>
<td>10N·s/m</td>
</tr>
<tr>
<td>$f_n$</td>
<td>0.53 Hz</td>
</tr>
</tbody>
</table>

Table 1: Material parameters of vertical seismic isolator.

In this case, Figure 6 shows the calculated acceleration response of the top and base plates as two examples of when the shaking frequency is changed. Here, $freq$ is the shaking frequency. Additionally, a reduced rate of acceleration that is a fraction of the base and top plates was used as an evaluation index of reduced acceleration. Figure 7 shows the calculated value of the reduced rate of acceleration.

(a) $a_0 = 1.2 \text{ m/s}^2$, $freq = 1 \text{ Hz}$  
(b) $a_0 = 1.2 \text{ m/s}^2$, $freq = 5 \text{ Hz}$

Figure 6. Calculated results of acceleration of top and base plate.
3.2 Calculation of seismic wave

The acceleration response of the top plate was calculated when a seismic wave was inputted to the base plate. For example, Figure 8 shows the calculated result of shaking for the UD (up and down) wave of the 2004 Niigata Prefecture Chuetsu Earthquake. The response acceleration was considerably reduced in regard to the input acceleration.

Additionally, in order to statistically evaluate the performance of the seismic isolator, we calculated the top plate’s acceleration responses when six seismic waves were inputted to the base plate. Table 2 summarizes the magnitude and earthquake focal depth for six seismic waves. Figure 9 shows the maximum acceleration of the top and base plates on several seismic waves, and Table 3 shows the reduced rate of acceleration on several seismic waves. The average reduced rate of acceleration was 0.32 for the six seismic waves. Although the reduced rate of acceleration, which has a frequency element close to the natural frequency of the seismic isolator, increased, the reduced rate of acceleration during the Hyogoken-Nanbu (Hanshin/Awaji) Earthquake is the maximum and the seismic isolation effect was sufficiently verified. In fact, it appears that the proposed vertical seismic isolator is valid. By adjusting offset $h$, the acceleration can be reduced for many seismic waves.
Figure 9. Maximum value of acceleration by using several seismic waves.

<table>
<thead>
<tr>
<th>Earthquake name</th>
<th>Magnitude</th>
<th>Earthquake focal depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tohoku(2011)</td>
<td>9.0</td>
<td>24km</td>
</tr>
<tr>
<td>Noto(2007)</td>
<td>6.9</td>
<td>11km</td>
</tr>
<tr>
<td>Chuetsuoki(2007)</td>
<td>6.8</td>
<td>17km</td>
</tr>
<tr>
<td>Chuetsu(2004)</td>
<td>6.8</td>
<td>13km</td>
</tr>
<tr>
<td>Tottori(2000)</td>
<td>7.3</td>
<td>9km</td>
</tr>
<tr>
<td>Hyogo(1995)</td>
<td>7.3</td>
<td>16km</td>
</tr>
</tbody>
</table>

Table 2: The scale of several seismic waves.

<table>
<thead>
<tr>
<th>Earthquake name</th>
<th>Reduced rate of acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tohoku(2011)</td>
<td>0.18</td>
</tr>
<tr>
<td>Noto(2007)</td>
<td>0.21</td>
</tr>
<tr>
<td>Chuetsuoki(2007)</td>
<td>0.52</td>
</tr>
<tr>
<td>Chuetsu(2004)</td>
<td>0.25</td>
</tr>
<tr>
<td>Tottori(2000)</td>
<td>0.20</td>
</tr>
<tr>
<td>Hyogo(1995)</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 3: The reduced rate of acceleration using several seismic waves.

3.3 Experiment device
We evaluated seismic isolation performance while inputting the base plate of the vertical seismic isolator by forced displacement. Figure 10 shows the experimental set up for the shaking test. The vibration exciter used in this study consisted of two hydraulic actuators, a 1.2 × 1.2 m shaking table, and hydraulic units. This exciter operates in both the horizontal and vertical directions. The frequency and acceleration of the input vibration were controlled by a PC connected to this two-dimensional shaker. The experiment was run when the vertical seismic isolator was attached to this vibration exciter. The experimental scenery is shown in Figure 11. A three-dimension acceleration meter (Wireless Technologies, Inc. WAA-006) was used in order to measure acceleration.
3.4 Sinusoidal wave experiment

We verified the acceleration response of the top plate when 1.2 m/s\(^2\) acceleration of the targeted frequency range was inputted. As one example, Figure 12 shows the result for sine 4 Hz. Table 4 shows the reduced rates of acceleration in sine 2, 3, and 4 Hz. The strain of the inputted wave is a characteristic of the vibration exciter. In Figure 12, the reduced acceleration response was confirmed. In fact, the reduced rate of acceleration was 0.57 when a value of sine 4 Hz was inputted to the base plate, thus confirming the seismic isolation effect of the proposed vertical seismic isolator. Furthermore, the reduced rate of acceleration was approximately 0.57 with sine 2 Hz and 3 Hz inputted to the base plate.

Additionally, the friction and the backlash of this device prevented the experiment from being run in the 0.1-1 Hz frequency range. It appears that the effect of rotational friction arising from the link pin and crank pin is significant and the magnification ratio of the acceleration response was consistently nearly one in low-frequency vibrations. Although the response may be changed at every frequency when the acceleration is increased, there is a limitation for the vibration exciter in increasing its acceleration. It is believed that using a bearing in the rotating part of the link or crank can reverse the effect of the rotational friction.
Figure 12. Experimental result of acceleration of top and base plate in sine 4Hz.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Reduced rate of acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>2Hz</td>
<td>0.58</td>
</tr>
<tr>
<td>3Hz</td>
<td>0.57</td>
</tr>
<tr>
<td>4Hz</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 4: The reduced rates of acceleration in sine 2, 3, and 4Hz.

4. CONCLUSION

In this study, in order to solve the problem of balancing downsizing the vertical seismic isolator and securing its stroke while reducing its natural frequency, we proposed a vertical seismic isolator that uses a link and crank. We can reduce the height of the vertical seismic isolator up to 20 cm and acquire ±10 cm stroke because it can support a vertical load using a horizontal spring only. As a result of the performance of this vertical seismic isolator as verified in the experiment and calculation, we can confirm the effect of seismic isolation for several seismic waves. The average reduced rate of acceleration in several seismic waves is 0.32. Additionally, as a result of the measurements of the top plate of the vertical seismic isolator when 2, 3, and 4 Hz sine waves were inputted to the base plate, we confirmed the effect of seismic isolation because the reduced rate of acceleration was 0.57.

We uncovered a problem with the experimental device involving a piece of the vertical seismic isolator needed to put it into practical use. We believe that this effect of rotational friction is reversible by using a bearing in the rotating part of the link or crank. Therefore, we will create a test model of the vertical seismic isolator to address this problem in a future study.

REFERENCES


